Tutorial No 3, Semester 1, 2024/25

1. A string which is vibrating with a frequency of 1,800 Hz has 9 antinodes between its two ends. A second string which has 6 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,050 Hz and is 60 cm long. Calculate the length of the first string which is vibrating with 9 antinodes. If a third string which is 80 cm long is vibrating at a frequency of 675 Hz, how many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

2. A kayak (small boat) is paddling along the surface of the sea in the same direction and the same speed as the water waves on the surface of the sea. The total length of the kayak is exactly equal to 5 complete wavelengths of the sea waves which are moving with a speed of 1 metre per second with a frequency of 1.25 Hz. What is the length of the kayak? The frequency of the waves then increases to 1.5 Hz and the speed of the waves increases to 1.5 metre per second. Calculate the number of wavelengths of the waves which would exactly equal the length of the kayak when this happens.

3. A string which has a fundamental frequency of 180 Hz is vibrating with 5 antinodes between its two

ends, and its frequency is the same as that of a closed pipe of length p cm which is vibrating with 4 nodes between its two ends (not counting the node at one end). Calculate the fundamental frequency of the closed pipe. When the closed pipe vibrates with 6 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 4 antinodes between its two ends (not counting the antinodes at both ends). Calculate the length of the open pipe.

4. A string which is vibrating with 5 nodes (not counting the nodes at both ends) is 30 cm long. The note produced by this string combines with a note from a closed pipe which has a fundamental frequency of 120 Hz to produce beats of 12 Hz. The closed pipe is vibrating with 3 nodes between its two ends (not counting the node at one end). When the string is then slightly shortened, the beat frequency increases (without passing through 0 Hz). What is the fundamental frequency of the string? If the string is then shortened from 30 cm to 28.4 cm, and assuming that the beats are still produced by the same harmonics of the string and the closed pipe as before, calculate the new beat frequency. If the length of the closed pipe is increased to 120% of its original length, calculate what the beat frequency would then be, assuming that the string is still 32.7 cm long.

5. An electronic tuner which is producing a musical note with a frequency of 220 Hz is used by a 'cellist in tuning her 'cello's A string. When the note from

the A string combines with the note from the tuner, beats of 6 Hz are heard. When the 'cellist gradually tightens the A string of the 'cello, the beat frequency gradually decreases (without passing through 0 Hz) to 4 Hz. What was the frequency of the note produced by the 'cello's A string when the beat frequency was equal to 6 Hz? To make the frequency of the A string come as close as possible to 220 Hz, what should the 'cellist do? If the beat frequency had increased to 7 Hz instead of decreasing when the string was tightened, what would the A string's frequency have been when the beat frequency was 6 Hz?

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly 3/2 as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?