Tutorial No 3, Semester 2, 2024/25

1. A string which has 8 antinodes between its two ends is vibrating with a frequency of 1,680 Hz. A second string 42 cm long which has 6 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,260 Hz. What is the length of the first string which is vibrating with 8 antinodes? If a third string which is 60 cm long is vibrating at a frequency of 1,134 Hz, how many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

2. A sailing boat is moving along in the same direction and the same speed as the sea waves on the surface of the sea, and the total length of the boat is exactly equal to 6 complete wavelengths of the sea waves. If the waves are moving with a speed of 0.9 metre per second with a frequency of 1 Hz, what is the length of the boat? If the speed of the waves then decreases to 0.5 metre per second and the frequency decreases to 0.8 Hz, what is the number of wavelengths of the waves which would exactly equal the length of the boat?

3. A string vibrating with 4 antinodes between its two ends has a fundamental frequency of 220 Hz. Its frequency is the same as that of a closed pipe of length k cm which is vibrating with 5 nodes between its two

ends (not counting the node at one end). What is the fundamental frequency of the closed pipe? When the closed pipe vibrates with 7 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 7 antinodes between its two ends (not counting the antinodes at both ends). What is the length of the open pipe?

4. A 66 cm string is vibrating with 6 nodes (not counting the nodes at both ends). The musical note produced by this string combines with a closed pipe which has a fundamental frequency of 98 Hz and is vibrating with 4 nodes between its two ends (not counting the node at one end). Beats of 14 Hz are produced, and when the string is then shortened slightly, the beat frequency increases (without passing through 0 Hz). Calculate the fundamental frequency of the string. If the string is shortened from 66 cm to 64 cm, and assuming that the beats are still produced by the same harmonics of the string and the closed pipe as before, what would be the new beat frequency? If the length of the closed pipe is decreased to 80% of its original length, what would the beat frequency then be, assuming that the string is still 64 cm long.

5. A viola player is tuning her viola's A string using an electronic tuner which is producing a musical note with a frequency of 440 Hz. Beats of 8 Hz are heard when the note from the A string combines with the note from the tuner. When the viola player gradu-

ally loosens the A string of the viola, the beat frequency gradually decreases (without passing through 0 Hz) to 6 Hz. What was the frequency of the note produced by the viola's A string when the beat frequency was equal to 8 Hz? What should the viola player do in order to make the frequency of the A string come as close as possible to 440 Hz? If the beat frequency had increased to 9 Hz instead of decreasing when the string was loosened, calculate the A string's frequency when the beat frequency was 8 Hz.

6. A closed pipe labelled A which has a fundamental frequency f Hz is sliced into nine pieces of equal lengths, creating eight short open pipes labelled A1 to A8, and a short closed pipe A9. Three of the short open pipes A6, A7 and A8 are joined up with A9 to make a closed pipe labelled B. The remaining short open pipes A1 to A5 are joined up to make an open pipe labelled C. What are the fundamental frequencies of the pipes A9, B and C? Calculate the ratio of the interval between the frequency of B when it vibrates with 6 nodes between its two ends (not counting the node at one end), and the frequency of C when it vibrates with 11 nodes between its two ends.

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly $3/2$ as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?