Answers to Tutorial No 4, Semester 1, 2024/25

1. A closed pipe labelled K has a fundamental frequency p Hz and is sliced into eight pieces of equal length. This creates seven short open pipes labelled K1 to K7, and a short closed pipe K8. Two of the short open pipes K6 and K7 are joined up with K8 to make a closed pipe labelled L, and the remaining short open pipes K1 to K5 are joined up to make an open pipe labelled M. Calculate the fundamental frequencies of the pipes K8, L and M. What is the ratio of the interval between the frequency of L when it vibrates with 5 nodes between its two ends (not counting the node at one end), and the frequency of M when it vibrates with 10 nodes between its two ends?

   Answer: Since the short closed pipe K8 is one-eighth the length of K, its fundamental frequency is equal to $8p$ Hz. The closed pipe L is three times the length of K8, so its fundamental frequency will be equal to $\frac{8p}{3}$ Hz. When L has 5 nodes, it will be at its 11th harmonic and its frequency will be equal to $\frac{8p}{3}$ Hz times 11 i.e. $\frac{88p}{3}$ Hz. A closed pipe which has a length five times the length of K8 will have a fundamental frequency of $\frac{8p}{5}$ Hz, so M which is an open pipe of the same length will have a fundamental frequency double this i.e. $\frac{16p}{5}$ Hz. When M has 10 nodes it will be at its 10th harmonic and its frequency will then be equal to $\frac{16p}{5}$ Hz times 10 i.e. $\frac{160p}{5}$

or $32p$ Hz. The ratio of the interval between $32p$ Hz and $\frac{88p}{3}$ Hz is given by $32p$ Hz divided by $\frac{88p}{3}$ Hz i.e. $\frac{12}{11}$.

2. An open pipe which is vibrating with 7 nodes between its two ends has a fundamental frequency of 180 Hz. When the note from the open pipe combines with the note from a closed pipe which is vibrating with 4 nodes between its two ends (not counting the node at one end), beats of 27 Hz are produced. What are the possible values of the fundamental frequency of the closed pipe? The closed pipe is then slightly shortened and the beat frequency is heard to decrease. Explain how the fundamental frequency of the closed pipe can be determined by the change in the beat frequency. Calculate the length of the open pipe if the length of the closed pipe before it was shortened was g cm.

   Answer: Since the open pipe has 7 nodes it is at its 7th harmonic and its frequency is given by 180 Hz times 7 i.e. 1,260 Hz. The beat frequency is 27 Hz, so the frequency of the closed pipe is either 1,260 Hz minus 27 Hz i.e. 1,233 Hz, or 1,260 Hz plus 27 Hz i.e. 1,287 Hz. The closed pipe has 4 nodes so it must be at its 9th harmonic, and hence its fundamental frequency is either 1,233 Hz divided by 9 i.e. 137 Hz, or 1,287 Hz divided by 9 i.e. 143 Hz. When the closed pipe is shortened, its frequency will increase. If the beat frequency decreases, the closed pipe's frequency must have moved nearer to that of the open pipe. The closed pipe's frequency thus must have been initially lower than that of the open pipe, i.e. it was equal to 1,233 Hz, and thus its fundamental frequency is 137 Hz. Since an open pipe of the same length would have a fundamental frequency double this i.e. 274 Hz, the length of the open pipe is given by g cm times $\frac{274}{180}$ i.e. $\frac{137g}{90}$ cm.

3. One theory of consonance or dissonance says that the degree of consonance between any two notes is dependent on the number of harmonics of one note which coincide with the harmonics of the other note. With this theory of consonance and dissonance, compare the consonance of a 120 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 120 Hz note for the comparison.)

   (a) A Just third.
   (b) A Just fifth.
   (c) A Just sixth.
   (d) A Just seventh.

   Answer: The first 18 harmonics of the 120 Hz note are: 120 Hz, 240 Hz, 360 Hz, 480 Hz, 600 Hz, 720 Hz, 840 Hz, 960 Hz, 1,080 Hz, 1,200 Hz, 1,320 Hz, 1,440 Hz, 1,560 Hz, 1,680 Hz, 1,800 Hz, 1,920 Hz, 2,040 Hz and 2,160 Hz. All the harmonics of the higher note for each of the above intervals are listed below, with the harmonics which will coincide with a harmonic of the 120 Hz note highlighted in **bold**:

   (a) A Just third above 120 Hz equals 120 Hz times $\frac{5}{4}$ i.e. 150 Hz. Harmonics: 150 Hz, 300 Hz, 450 Hz,

600 Hz Hz, 750 Hz, 900 Hz, 1,050 Hz, **1,200 Hz**, 1,350 Hz, 1,500 Hz, 1,650 Hz, **1,800 Hz**, 1,950 Hz and 2,100 Hz.

(b) A Just fifth above 120 Hz equals 120 Hz times $\frac{3}{2}$ i.e. 180 Hz. Harmonics: 180 Hz, **360 Hz** Hz, 540 Hz, **720 Hz** Hz, 900 Hz, **1,080 Hz** Hz, 1,260 Hz, **1,440 Hz**, 1,620 Hz, **1,800 Hz** Hz, 1,980 Hz, and **2,160 Hz**.

(c) A Just sixth above 120 Hz equals 120 Hz times $\frac{5}{3}$ i.e. 200 Hz. Harmonics: 200 Hz, 400 Hz, **600 Hz** Hz, 800 Hz, 1,000 Hz, **1,200 Hz** Hz, 1,400 Hz, 1,600 Hz, **1,800 Hz** Hz, 2,000 Hz, and 2,200 Hz.

(d) A Just seventh above 120 Hz equals 120 Hz times $\frac{15}{8}$ i.e. 225 Hz. Harmonics: 225 Hz, 450 Hz, 675 Hz, 900 Hz, 1,125 Hz, 1,350 Hz, 1,575 Hz, **1,800 Hz** and 2,025 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant are the Just third and Just sixth with three harmonics coinciding. The least consonant is the Just seventh with only one harmonic coinciding.

4. The action of a Cristofori piano has three levers for each of its keys which cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The upwards velocity of the hammer is equal to the movement of the downwards key multiplied by the first, second and third levers by factors of 1, 1.8 and 4.2

times respectively. What is the upwards velocity of the hammer when the downwards velocity of the corresponding key is 2.4 cm per second? The second lever of the action is then replaced with a new lever which has a multiplication factor different from 1.8 times, and after the new lever has been installed, a downwards velocity of the key of 2.5 cm per second is required to give the same upwards velocity of the hammer as before. Calculate the multiplication factor of the new second lever.

Answer: Since the three levers when acting together will have a combined multiplication factor equal to 1 times 1.8 times 4.2 i.e. 7.56, the upwards velocity of the hammer is equal to 2.4 cm per second times 7.56 i.e. 18.144 cm per second. A downwards key velocity of 2.5 cm per second is required for the same upwards velocity of the hammer after the new second lever has been installed, so the new combined multiplication factor for the three levers acting together is equal to 18.144 cm per second divided by 2.5 cm per second i.e. 7.2576. The multiplication factor of the new second lever is thus equal to 7.2576 divided by 4.2, giving a multiplication factor of 1.728 times for the new second lever.

5. The soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal on a particular grand piano are functioning normally. A pianist plays on this piano, and each of the following four different situations regarding the pedals may occur. In situation 1, the sostenuto pedal is depressed, then the notes

C2 and E2 are depressed, and then these keys are released. If she keeps pressing the sostenuto pedal will the notes C2 and E2 be sustained? In situation 2, the notes C2 and E2 are depressed, then the sostenuto pedal is depressed, and then the C2 and E2 keys are released while the sostenuto pedal remains depressed. Will the notes C2 and E2 be sustained? In situation 3, the sustain pedal is depressed, then the keys C2 and E2 are depressed and then released, keeping the sustain pedal depressed. Will the C2 and E2 notes be kept sustained? In situation 4, the the keys C2 and E2 are depressed, then the sustain pedal is depressed and then the keys C2 and E2 are released, and then the sustain pedal is kept depressed. Will the C2 and E2 notes be sustained in this case? **Answer:** If the sostenuto pedal is depressed after the keys have been depressed, the notes C2 and E2 will be sustained even after their keys have been released, as long as the sostenuto pedal is kept depressed. If the sostenuto pedal is depressed before the C2 and E2 keys have been depressed, this will not happen. Therefore the notes will be sustained only in situation 2 but not in situations 1. If the sustain pedal is depressed before or while the keys C2 and E2 are depressed, the notes C2 and E2 will be sustained as long as the sustain pedal is depressed. Therefore the notes C2 and E2 will be sustained in both situations 3 and 4.

Scientific Inquiry discussion points

(a) It has now been ascertained, by making scientific observations and from the technical knowledge of how the piano action works, that a pianist playing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontrovertible proof of the earth's roundness. A more recent and current example is climate change. While the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due *to man-made causes has become strong enough to be accepted by the great majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.*