Answers to Tutorial No 4, Semester 2, 2024/25

1. An open pipe labelled P is sliced into seven pieces of equal lengths, creating seven short open pipes labelled P1 to P7. Two of the short open pipes P1 and P2 are joined up to make a pipe labelled Q. The three short open pipes P3, P4 and P5 are joined up to make another open pipe labelled R. The remaining two short open pipes are joined up and one end closed up to make a closed pipe labelled S. All three pipes are then made to vibrate at their fundamental frequencies. Arrange the following frequencies in order of increasing frequency: the second harmonic of Q, the fourth harmonic of R and the fifth harmonic of S.

   Answer: Let the fundamental frequency of P be f Hz. The fundamental frequency of Q is then equal to f Hz times $\frac{7}{2}$ i.e. $\frac{7f}{2}$ Hz and its second harmonic is $\frac{7f}{2}$ Hz times 2 i.e. $\frac{14f}{2}$ Hz or $7f$ Hz. Likewise, the fundamental frequency of R is equal to f Hz times $\frac{7}{3}$ i.e. $\frac{7f}{3}$ Hz and its fourth harmonic is $\frac{7f}{3}$ Hz times 4 i.e. $\frac{28f}{3}$ Hz. An open pipe with the same length as S will have a fundamental frequency equal to f Hz times $\frac{7}{2}$ i.e. $\frac{7f}{2}$ Hz. A closed pipe of the same length will have a fundamental frequency half of this i.e. $\frac{7f}{4}$ Hz, and its fifth harmonic will be equal to $\frac{7f}{4}$ Hz times 5 i.e. $\frac{35f}{4}$ Hz. There the three frequencies in increasing order of frequency are: the second harmonic of Q, the fifth harmonic of S, and the fourth harmonic of R.

2. An open pipe which has a fundamental frequency of 210 Hz is vibrating with 8 nodes between its two ends, and when the note from the open pipe combines with the note from a closed pipe which is vibrating with 3 nodes between its two ends (not counting the node at one end), beats of 14 Hz are produced. Calculate the possible values of the fundamental frequency of the closed pipe. When the closed pipe is slightly shortened, the beat frequency is heard to increase. Show how the fundamental frequency of the closed pipe can be determined by the change in the beat frequency. What is the length of the open pipe if the length of the closed pipe before it was shortened was h cm?

   Answer: The open pipe has 8 nodes so it is at its 8th harmonic and its frequency is given by 210 Hz times 8 i.e. 1,680 Hz. Since the beat frequency is 14 Hz, the frequency of the closed pipe is either 1,680 Hz minus 14 Hz i.e. 1,666 Hz, or 1,680 Hz plus 14 Hz i.e. 1,694 Hz. The closed pipe has 3 nodes so it must be at its 7th harmonic. Therefore its fundamental frequency is either 1,666 Hz divided by 7 i.e. 238 Hz, or 1,694 Hz divided by 7 i.e. 242 Hz. On shortening the closed pipe, its frequency will increase so if the beat frequency increases, the closed pipe's frequency must have moved further from that of the open pipe. Therefore the closed pipe's frequency must have been initially higher than that of the open pipe, i.e. it was equal to 1,694 Hz, and its fundamental frequency is 242 Hz. An open pipe of the same length would have a fundamental frequency double this i.e. 484 Hz, so the length of the open pipe is given by h cm times $\frac{484}{210}$ i.e. $\frac{242h}{105}$ cm.

3. The degree of consonance between any two notes is dependent on the number of harmonics of one note which coincide with the harmonics of the other note, according to one theory of consonance. With this theory of consonance, compare the consonance of a 150 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 150 Hz note for the comparison.)

   (a) A Just second.
   (b) A Just third.
   (c) A Just fifth.
   (d) A Just sixth.

   Answer: The first 18 harmonics of the 150 Hz note are: 150 Hz, 300 Hz, 450 Hz, 600 Hz, 750 Hz, 900 Hz, 1,050 Hz, 1,200 Hz, 1,350 Hz, 1,500 Hz, 1,650 Hz, 1,800 Hz, 1,950 Hz, 2,100 Hz, 2,250 Hz, 2,400 Hz, 2,550 Hz and 2,700 Hz. All the harmonics of the higher note for each of the above intervals are listed below, with the harmonics which actually coincide with a harmonic of the 150 Hz note highlighted in **bold**:

   (a) A Just second above 150 Hz equals 150 Hz times $\frac{9}{8}$ i.e. 168.75 Hz. Harmonics: 168.75 Hz, 337.5

Hz, 506.25 Hz, 675 Hz, 843.75 Hz, 1,012.5 Hz, 1,181.25 Hz, **1,350 Hz**, 1,518.75 Hz, 1,687.5 Hz, 1,856.25 Hz, 2,025 Hz, 2,193.75 Hz, 2,362.5 Hz, 2,531.25 Hz and **2,700 Hz**.

(b) A Just third above 150 Hz equals 150 Hz times $\frac{5}{4}$ i.e. 187.5 Hz. Harmonics: 187.5 Hz, 375 Hz, 562.5 Hz, **750 Hz** Hz, 937.5 Hz, 1,125 Hz, 1,312.5 Hz, **1,500 Hz**, 1,687.5 Hz, 1,875 Hz, 2,062.5 Hz, **2,250 Hz**, 2,437.5 Hz and 2,625 Hz.

(c) A Just fifth above 150 Hz equals 150 Hz times $\frac{3}{2}$ i.e. 225 Hz. Harmonics: 225 Hz, **450 Hz** Hz, 675 Hz, **900 Hz** Hz, 1,125 Hz, **1,350 Hz** Hz, 1,575 Hz, **1,800 Hz**, 2,025 Hz, **2,250 Hz** Hz, 2,475 Hz, and **2,700 Hz**.

(d) A Just sixth above 150 Hz equals 150 Hz times $\frac{5}{3}$ i.e. 250 Hz. Harmonics: 250 Hz, 500 Hz, **750 Hz** Hz, 1,000 Hz, 1,250 Hz, **1,500 Hz** Hz, 1,750 Hz, 2,000 Hz, **2,250 Hz** Hz, 2,500 Hz, and 2,750 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant are the Just third and Just sixth with three harmonics coinciding. The least consonant is the Just second with only two harmonics coinciding.

4. A Cristofori piano has an action with three levers for each of its keys which cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The upwards velocity of the hammer is given by the movement of the downwards key multiplied by the first,

second and third levers by factors of 1, 2 and 4.5 times respectively. Calculate the upwards velocity of the hammer when the downwards velocity of the corresponding key is 2.5 cm per second. After the second lever of the action is replaced with a new lever which has a multiplication factor different from 2 times, a downwards velocity of the key of 2 cm per second is required to give the same upwards velocity of the hammer as before. What is the multiplication factor of the new second lever?

Answer: The three levers when acting together will have a combined multiplication factor equal to 1 times 2 times 4.5 i.e. 9.0, so the upwards velocity of the hammer is equal to 2.5 cm per second times 9 i.e. 22.5 cm per second. After a new second lever has been installed, a downwards key velocity of 2 cm per second is required for the same upwards velocity of the hammer. Therefore the new combined multiplication factor for the three levers acting together is equal to 22.5 cm per second divided by 2 cm per second i.e. 11.25. The multiplication factor of the new second lever is thus equal to 11.25 divided by 4.5, giving a multiplication factor of 2.5 times for the new second lever.

5. On a certain grand piano, the soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal are functioning normally. When a pianist plays on this piano, each of the following four different situations regarding the pedals may occur. In situation 1, the sustain pedal is depressed, then the keys A3 and

E4 are depressed and then released, keeping the sustain pedal depressed. Will the A3 and E4 notes be kept sustained? In situation 2, the sostenuto pedal is depressed, then the notes A3 and E4 are depressed, and then these keys are released. If the pianist keeps pressing the sostenuto pedal will the notes A3 and E4 be sustained? In situation 3, the notes A3 and E4 are depressed, then the sostenuto pedal is depressed, and then the A3 and E4 keys are released while the sostenuto pedal remains depressed. Will the notes A3 and E4 be sustained? In situation 4, the keys A3 and E4 are depressed, then the soft pedal is depressed and then the keys A3 and E4 are released, with the soft pedal kept depressed. Will the A3 and E4 notes be sustained in this case?

Answer: If the sustain pedal is depressed before or while the keys A3 and E4 are depressed, the notes A3 and E4 will be sustained as long as the sustain pedal is depressed, and therefore the notes A3 and E4 will be sustained in situation 1. If the sostenuto pedal is depressed after the keys have been depressed, the notes A3 and E4 will be sustained even after their keys have been released, as long as the sostenuto pedal is kept depressed. If the sostenuto pedal is depressed before the A3 and E4 keys have been depressed, this will not happen. Therefore the notes will be sustained only in situation 3 but not in situation 2. The soft pedal has no effect on the sustaining of the piano notes, so in situation 4 the notes will not be sustained.

Scientific Inquiry discussion points

(a) It has now been ascertained, by making scientific observations and from the technical knowledge of how the piano action works, that a pianist playing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontrovertible proof of the earth's roundness. A more recent and current example is climate change. While *the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due to man-made causes has become strong enough to be accepted by the great majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.*