Answers to Tutorial No 4, Semester 1, 2025/26

1. An open pipe labelled E is sliced into nine pieces of equal lengths, creating nine short open pipes labelled E1 to E9, and five of the short open pipes E1, E2, E3, E4 and E5 are joined up to make a pipe labelled F. The three short open pipes E6, E7 and E8 are joined up to make another open pipe labelled G. The remaining short open pipe E9 is closed up at one end to make a closed pipe labelled H. All three pipes F, G and H are then made to vibrate at their fundamental frequencies. Arrange the following frequencies in order of increasing frequency: the fourth harmonic of F, the second harmonic of G and the third harmonic of H.

   Answer: Let the fundamental frequency of E be f Hz. The fundamental frequency of F is then equal to f Hz times $\frac{9}{5}$ i.e. $\frac{9f}{5}$ Hz and its fourth harmonic is $\frac{9f}{5}$ Hz times 4 i.e. $\frac{36f}{5}$ Hz. The fundamental frequency of G is likewise equal to f Hz times $\frac{9}{3}$ i.e. $3f$ Hz and its second harmonic is $3f$ Hz times 2 i.e. $6f$ Hz. An open pipe with the same length as H will have a fundamental frequency equal to f Hz times 9 i.e. $9f$ Hz. A closed pipe of the same length will have a fundamental frequency half of this i.e. $\frac{9f}{2}$ Hz, and its third harmonic will be equal to $\frac{9f}{2}$ Hz times 3 i.e. $\frac{27f}{2}$ Hz. Therefore the three frequencies in order of increasing frequency are: the second harmonic of G,

the fourth harmonic of F, and the third harmonic of H.

2. An open pipe vibrating with 7 nodes between its two ends has a fundamental frequency of 180 Hz. When the note from the open pipe combines with the note from a closed pipe which is vibrating with 4 nodes between its two ends (not counting the node at one end), beats of 9 Hz are produced. What are the possible values of the fundamental frequency of the closed pipe? If the beat frequency is heard to decrease when the closed pipe is slightly shortened, show how we can find out the fundamental frequency of the closed pipe by the change in the beat frequency. Calculate the length of the open pipe if the length of the closed pipe before it was shortened was k cm.

   Answer: Since the open pipe has 7 nodes it is at its 7th harmonic and its frequency is equal to 180 Hz times 7 i.e. 1,260 Hz. The beat frequency is 9 Hz, so the frequency of the closed pipe is either 1,260 Hz minus 9 Hz i.e. 1,251 Hz, or 1,260 Hz plus 9 Hz i.e. 1,269 Hz. The closed pipe has 4 nodes so it must be at its 9th harmonic, and thus its fundamental frequency is either 1,251 Hz divided by 9 i.e. 139 Hz, or 1,269 Hz divided by 9 i.e. 141 Hz. When the closed pipe is shortened, its frequency will increase so if the beat frequency decreases, the closed pipe's frequency must have moved nearer to that of the open pipe. The closed pipe's frequency thus must have been initially lower than that of the open pipe, i.e.

it was equal to 1,251 Hz, and its fundamental frequency is 139 Hz. An open pipe of the same length would have a fundamental frequency double this i.e. 278 Hz, so the length of the open pipe is equal to k cm times $\frac{278}{180}$ i.e. $\frac{139k}{90}$ cm.

3. According to one theory of consonance, the degree of consonance between any two notes depends on the number of harmonics of one note which coincide with the harmonics of the other note. Using this theory of consonance, compare the consonance of a 200 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 200 Hz note for the comparison.)

   (a) A Just second.

   (b) A Just third.

   (c) A Just fifth.

   (d) A Just seventh.

   Answer: The first 18 harmonics of the 200 Hz note are: 200 Hz, 400 Hz, 600 Hz, 800 Hz, 1,000 Hz, 1,200 Hz, 1,400 Hz, 1,600 Hz, 1,800 Hz, 2,000 Hz, 2,200 Hz, 2,400 Hz, 2,600 Hz, 2,800 Hz, 3,000 Hz, 3,200 Hz, 3,400 Hz and 3,600 Hz. All the harmonics of the higher note for each of the above intervals are listed below, with the harmonics which actually coincide with a harmonic of the 200 Hz note highlighted in **bold**:

   (a) A Just second above 200 Hz equals 200 Hz times $\frac{9}{8}$ i.e. 225 Hz. Harmonics: 225 Hz, 450 Hz, 675

Hz, 900 Hz, 1,125 Hz, 1,350 Hz, 1,575 Hz, **1,800 Hz**, 2,025 Hz 2,250 Hz, 2,475 Hz, 2,700 Hz, 2,925 Hz, 3,150 Hz, 3,375 and **3,600 Hz**.

(b) A Just third above 200 Hz equals 200 Hz times $\frac{5}{4}$ i.e. 250 Hz. Harmonics: 250 Hz, 500 Hz, 750 Hz, **1,000 Hz** Hz, 1,250 Hz, 1,500 Hz, 1,750 Hz, **2,000 Hz**, 2,250 Hz, 2,500 Hz, 2,750 Hz, **3,000 Hz**, 3,250 Hz and 3,500 Hz.

(c) A Just fifth above 200 Hz equals 200 Hz times $\frac{3}{2}$ i.e. 300 Hz. Harmonics: 300 Hz, **600 Hz** Hz, 900 Hz, **1,200 Hz** Hz, 1,500 Hz, **1,800 Hz** Hz, 2,100 Hz, **2,400 Hz**, 2,700 Hz, **3,000 Hz** Hz, 3,300 Hz, and **3,600 Hz**.

(d) A Just seventh above 200 Hz equals 200 Hz times $\frac{15}{8}$ i.e. 375 Hz. Harmonics: 375 Hz, 750 Hz, 1,125 Hz, 1,500 Hz, 1,875 Hz, 2,250 Hz, 2,625 Hz, **3,000 Hz**, 3,375 Hz and 3,750 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant are the Just third and Just second with three and two harmonics coinciding respectively. The least consonant is the Just seventh with only one harmonic coinciding.

4. The action of a Cristofori piano has three levers for each of its keys which cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The upwards velocity of the hammer is given by the movement of the downwards key multiplied by the first,

second and third levers by factors of 1, 2 and 5 times respectively. Calculate the upwards velocity of the hammer when the downwards velocity of the corresponding key is 2.5 cm per second. After the second lever of the action has been replaced with a new lever which has a multiplication factor different from 2 times, a downwards velocity of the key of 2 cm per second is required to give the same upwards velocity of the hammer as before. Calculate the multiplication factor of the new second lever

Answer: The three levers when acting together will have a combined multiplication factor equal to 1 times 2 times 5 i.e. 10, so the upwards velocity of the hammer is equal to 2.5 cm per second times 10 i.e. 25 cm per second. After a new second lever has been installed, a downwards key velocity of 2 cm per second is required for the same upwards velocity of the hammer. Therefore the new combined multiplication factor for the three levers acting together is equal to 25 cm per second divided by 2 cm per second i.e. 12.5. The multiplication factor of the new second lever is thus equal to 12.5 divided by 5, i.e. 2.5 times for the new second lever.

5. The soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal on a particular grand piano are functioning as normal. When a pianist plays on this piano, each of the following four different situations regarding the pedals may occur. In situation 1, the notes G2 and D3 are depressed, then the sostenuto pedal is depressed, and then the G2

and D3 keys are released while the sostenuto pedal remains depressed. Will the notes G2 and D3 be sustained? In situation 2, the sostenuto pedal is depressed then the keys G2 and D3 are depressed and the keys G2 and D3 are released with the sostenuto pedal kept depressed. Will the G2 and D3 notes be sustained in this case? In situation 3, the sustain pedal is depressed, then the keys G2 and D3 are depressed and then released, keeping the sustain pedal depressed. Will the G2 and D3 notes be kept sustained? In situation 4, the notes G2 and D3 are depressed, then the sustain pedal is depressed, and then these keys are released. If the pianist keeps depressing the sustain pedal will the notes G2 and D3 be sustained?

Answer: If the sostenuto pedal is depressed after the keys have been depressed, the notes G2 and D3 will be sustained even after their keys have been released, as long as the sostenuto pedal is kept depressed. If the sostenuto pedal is depressed before the G2 and D3 keys have been depressed, this will not happen. Therefore the notes will be sustained only in situation 1 but not in situation 2. If the sustain pedal is depressed before or while the keys G2 and D3 are depressed, the notes G2 and D3 will be sustained as long as the sustain pedal is depressed, and therefore the notes G2 and D3 will be sustained in both situations 3 and 4.

Scientific Inquiry discussion points

(a) It has now been ascertained, by making scientific

observations and from the technical knowledge of how the piano action works, that a pianist playing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontrovertible proof of the earth's roundness. A more recent and current example is climate change. While the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due

to man-made causes has become strong enough to be accepted by the great majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.