Answers to Tutorial No 4, Semester 2, 2025/26

1. A closed pipe labelled A is sliced into eight pieces of equal lengths, creating seven short open pipes labelled A1 to A7, and one short closed pipe labelled A8. Three of the short open pipes A1, A2 and A3 are joined up to make an open pipe labelled B. The three pipes A4, A5 and A6 are joined up and one end closed up to make a closed pipe labelled C. The remaining short open pipe A7 is joined up with A8 to make a closed pipe labelled D. Arrange the following frequencies in order of increasing frequency: the second harmonic of B, the fifth harmonic of C and the third harmonic of D.

   Answer: Let the fundamental frequency of A be f Hz. The fundamental frequency of B is then equal to f Hz times $\frac{8}{3}$ times 2 i.e. $\frac{16f}{3}$ Hz and its second harmonic is $\frac{16f}{3}$ Hz times 2 i.e. $\frac{32f}{3}$ Hz. The fundamental frequency of C is equal to f Hz times $\frac{8}{3}$ and its fifth harmonic is $\frac{8f}{3}$ Hz times 5 i.e. $\frac{40f}{3}$ Hz. The fundamental frequency of D is f Hz times $\frac{8}{2}$ i.e. $4f$ Hz and its third harmonic is $4f$ Hz times 3 i.e. $12f$ Hz. Therefore the three frequencies in increasing order of frequency are: the second harmonic of B, the third harmonic of D and the fifth harmonic of C.

2. A closed pipe vibrating with 5 nodes between its two ends (not counting the node at one end) has a fundamental frequency of 80 Hz. When the note

from the closed pipe combines with the note from an open pipe which is vibrating with 4 nodes between its two ends, beats of 8 Hz are heard. Calculate the possible values of the fundamental frequency of the open pipe. If the beat frequency is heard to decrease when the open pipe is slightly shortened, how can we determine the fundamental frequency of the open pipe by the change in the beat frequency? Calculate the length of the closed pipe if the length of the open pipe before it was shortened was p cm.

Answer: The closed pipe has 5 nodes so it is at its 11th harmonic and its frequency is equal to 80 Hz times 11 i.e. 880 Hz. Since the beat frequency is 8 Hz, the frequency of the open pipe is either 880 Hz minus 8 Hz i.e. 872 Hz, or 880 Hz plus 8 Hz i.e. 888 Hz. The open pipe has 4 nodes so it must be at its 4th harmonic, and therefore its fundamental frequency is either 872 Hz divided by 4 i.e. 218 Hz, or 888 Hz divided by 4 i.e. 222 Hz. Its frequency will increase if the open pipe is shortened, so if the beat frequency decreases, the open pipe's frequency must have moved nearer to that of the closed pipe. Therefore the open pipe's frequency must have been initially lower than that of the closed pipe, i.e. it was equal to 872 Hz, and its fundamental frequency is 218 Hz. A closed pipe of the same length would have a fundamental frequency half this i.e. 109 Hz, so the length of the closed pipe is equal to p cm times $\frac{109}{80}$ i.e. $\frac{109p}{80}$ cm.

3. One theory of consonance says that the degree of

consonance between any two notes depends on the number of harmonics of one note which coincide with the harmonics of the other note. Using this theory of consonance, compare the consonance of a 120 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 120 Hz note for the comparison.)

(a) A Just second.
(b) A Just fourth.
(c) A Just fifth.
(d) A Just sixth

Answer: The first 18 harmonics of the 120 Hz note are: 120 Hz, 240 Hz, 360 Hz, 480 Hz, 600 Hz, 720 Hz, 840 Hz, 960 Hz, 1,080 Hz, 1,200 Hz, 1,320 Hz, 1,440 Hz, 1,560 Hz, 1,680 Hz, 1,800 Hz, 1,920 Hz, 2,040 Hz and 2,160 Hz. The harmonics of the higher note for each of the above intervals are listed below, with the harmonics which actually coincide with a harmonic of the 120 Hz note highlighted in **bold**:

(a) A Just second above 120 Hz equals 120 Hz times $\frac{9}{8}$ i.e. 135 Hz. Harmonics: 135 Hz, 270 Hz, 405 Hz, 540 Hz, 675 Hz, 810 Hz, 945 Hz, **1,080 Hz**, 1,215 Hz, 1,350 Hz, 1,485 Hz, 1,620 Hz, 1,755 Hz, 1,890 Hz, 2,025 Hz and **2,160 Hz**.
(b) A Just fourth above 120 Hz equals 120 Hz times $\frac{4}{3}$ i.e. 160 Hz. Harmonics: 160 Hz, 320 Hz, **480 Hz**, 640 Hz, 800 Hz, **960 Hz**, 1,120 Hz, 1,280 Hz. **1,440 Hz**, 1,600 Hz, 1,760 Hz, **1,920 Hz**, 2,080 Hz and 2,240 Hz.

(c) A Just fifth above 120 Hz equals 120 Hz times $\frac{3}{2}$ i.e. 180 Hz. Harmonics: 180 Hz, **360 Hz**, 540 Hz, **720 Hz** Hz, 900 Hz, **1,080 Hz** Hz, 1,260 Hz, **1,440 Hz**, 1,620 Hz, **1,800 Hz** Hz, 1,980 Hz, and **2,160 Hz**.

(d) A Just sixth above 120 Hz equals 120 Hz times $\frac{5}{3}$ i.e. 200 Hz. Harmonics: 200 Hz, 400 Hz, **600 Hz**, 800 Hz, 1,000 Hz, **1,200 Hz**, 1,400 Hz, 1,600 Hz, **1,800 Hz** and 2,000 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant are the Just fourth and Just sixth with four and three harmonics coinciding respectively. The least consonant is the Just second with two harmonics coinciding.

4. A Cristofori piano has an action with three levers for each of its keys which will cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The upwards velocity of the hammer is equal to the movement of the downwards key multiplied by the first, second and third levers by factors of 1, 2.4 and 4.8 times respectively. Calculate the upwards velocity of the hammer when the downwards velocity of the corresponding key is 3 cm per second. When the third lever of the action is replaced with a new lever with a multiplication factor different from 4.8 times, a downwards velocity of the key of 2.5 cm per second is required to give the same upwards velocity of the hammer as before. What is the multiplication factor

of the new third lever?

Answer: When the three levers act together they will have a combined multiplication factor equal to 1 times 2.4 times 4.8 i.e. 11.52, so the upwards velocity of the hammer is equal to 3 cm per second times 11.52 i.e. 34.56 cm per second. After a new third lever has been installed, a downwards key velocity of 2.5 cm per second is required for the same upwards velocity of the hammer. The new combined multiplication factor for the three levers acting together is hence given by 34.56 cm per second divided by 2.5 cm per second i.e. 13.824. Therefore the multiplication factor of the new third lever is equal to 13.824 divided by 2.4, i.e. 5.76 times.

5. On a certain grand piano, the soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal are functioning as normal. Each of the following four different situations regarding the use of the pedals may occur when a pianist plays on this piano. In situation 1, the sostenuto pedal is depressed then the keys A2 and E3 are depressed and the keys A2 and E3 are released with the sostenuto pedal kept depressed. Will the A2 and E3 notes be sustained? In situation 2, the notes A2 and E3 are depressed, then the sostenuto pedal is depressed, and then the A2 and E3 keys are released while the sostenuto pedal remains depressed. Will the notes A2 and E3 be sustained in this case? In situation 3, the sustain pedal is depressed, then the keys A2 and E3 are depressed and then released, keeping the sustain pedal

depressed. Will the A2 and E3 notes be kept sustained? In situation 4, the notes A2 and E3 are depressed, then the soft pedal is depressed, and then these keys are released. If the pianist keeps depressing the soft pedal will the notes A2 and E3 be sustained?

Answer: The notes A2 and G3 will be sustained even after their keys have been released, as long as the sostenuto pedal is kept depressed, if the sostenuto pedal is depressed after the keys have been depressed. If the sostenuto pedal is depressed before the A2 and E3 keys have been depressed, the notes will not be sustained. Hence the notes will be sustained only in situation 2 but not in situation 1. If the sustain pedal is depressed before or while the keys A2 and E3 are depressed, the notes A2 and E3 will be sustained as long as the sustain pedal is depressed, and therefore the notes A2 and E3 will be sustained in situation 3. The soft pedal does not lift up the dampers of the piano, so in situation 4 the notes A2 and E3 will not be sustained.

Scientific Inquiry discussion points

It has now been ascertained, by making scientific observations and from the technical knowledge of how the piano action works, that a pianist playing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontrovertible proof of the earth's roundness. A more recent and current example is climate change. While the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due to man-made causes has become strong enough to be accepted by the great majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.