Answers to Tutorial No 4, Semester 1, 2026/27

1. An open pipe labelled K is sliced into eight pieces of equal lengths to create eight short open pipes labelled K1 to K8. Two of the short open pipes K1 and K2 are joined up to make an open pipe labelled L. The pipes K3, K4 and K5 are joined up to make another open pipe labelled M. The remaining pipes K6, K7 and K9 are joined up and closed up at one end to make a closed pipe labelled N. All three pipes L, M and N are then made to vibrate at their fundamental frequencies. Arrange the following frequencies in order of increasing frequency: the third harmonic of L, the fifth harmonic of M and the seventh harmonic of N.

   Answer: Let the fundamental frequency of K be f Hz. The fundamental frequency of L is then given by f Hz times $\frac{8}{2}$ ie $4f$ Hz and its third harmonic is $12f$ Hz. The fundamental frequency of M is similarly equal to $\frac{8f}{3}$ Hz and its fifth harmonic is $\frac{8f}{3}$ Hz times 5 i.e. $\frac{40f}{3}$ Hz. An open pipe with the same length as N will have a fundamental frequency equal to $\frac{8f}{3}$Hz. A closed pipe of the same length will have a fundamental frequency half of this i.e. $\frac{4f}{3}$ Hz, and its seventh harmonic will be equal to $\frac{4f}{3}$ Hz times 7 i.e. $\frac{28f}{3}$ Hz. Therefore the three frequencies in order of increasing frequency are: the seventh harmonic of N, the third harmonic of L and the fifth harmonic of

M.

2. An open pipe which has a fundamental frequency of 140 Hz is vibrating with 6 nodes between its two ends, and when the note from the open pipe combines with the note from a closed pipe which is vibrating with 3 nodes between its two ends (not counting the node at one end), beats of 14 Hz are produced. What are the possible values of the fundamental frequency of the closed pipe? The beat frequency is then heard to increase when the closed pipe is slightly shortened. Explain how we can find out the fundamental frequency of the closed pipe by the change in the beat frequency. What is the length of the open pipe if the length of the closed pipe before it was shortened was p cm?

   Answer: The open pipe has 6 nodes so it is at its 6th harmonic and its frequency is given by 140 Hz times 6 i.e. 840 Hz. Since the beat frequency is 14 Hz, the frequency of the closed pipe is either 840 Hz minus 14 Hz i.e. 826 Hz, or 840 Hz plus 14 Hz i.e. 854 Hz. The closed pipe has 3 nodes so it must be at its 7th harmonic, and its fundamental frequency is therefore either 826 Hz divided by 7 i.e. 118 Hz, or 854 Hz divided by 7 i.e. 122 Hz. When the closed pipe is shortened its frequency will increase. So if the beat frequency increases, the closed pipe's frequency must have moved further from that of the open pipe, and the closed pipe's frequency thus must have been initially higher than that of the open pipe. Thus it was equal to 854 Hz, and its fundamental frequency

is 122 Hz. Since an open pipe of the same length would have a fundamental frequency double this i.e. 244 Hz, the length of the open pipe is equal to p cm times $\frac{244}{140}$ i.e. $\frac{61p}{35}$ cm.

3. According to one theory of consonance, the degree of consonance between any two notes depends on the number of harmonics of one note which coincide with the harmonics of the other note. Using this theory of consonance, compare the consonance of a 60 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 60 Hz note for the comparison.)

   (a) A Just third.

   (b) A Just fourth.

   (c) A Just fifth.

   (d) A Just seventh.

   Answer: The first 18 harmonics of the 60 Hz note are: 60 Hz, 120 Hz, 180 Hz, 240 Hz, 300 Hz, 360 Hz, 420 Hz, 480 Hz, 540 Hz, 600 Hz, 660 Hz, 720 Hz, 780 Hz, 840 Hz, 900 Hz, 960 Hz, 1,020 Hz and 1,080 Hz. All the harmonics of the higher note for each of the above intervals are listed below, with the harmonics which actually coincide with a harmonic of the 200 Hz note highlighted in **bold**:

   (a) A Just third above 60 Hz equals 60 Hz times $\frac{5}{4}$ i.e. 75 Hz. Harmonics: 75 Hz, 150 Hz, 225 Hz, **300 Hz** Hz, 375 Hz, 450 Hz, 525 Hz, **600 Hz**, 675 Hz, 750 Hz, 825 Hz, **900 Hz**, 975 Hz and and 1,050 Hz.

(b) A Just fourth above 60 Hz equals 60 Hz times $\frac{4}{3}$ i.e. 80 Hz. Harmonics: 80 Hz, 160 Hz, **240 Hz**, 320 Hz, 400 Hz, **480 Hz**, 560 Hz, 640 Hz, **720 Hz**, 800 Hz, 880 Hz,**960 Hz**, 1,040 Hz and 1,120 Hz.

(c) A Just fifth above 60 Hz equals 60 Hz times $\frac{3}{2}$ i.e. 90 Hz. Harmonics: 90 Hz, **180 Hz** Hz, 270 Hz, **360 Hz** Hz, 450 Hz, **540 Hz** Hz, 630 Hz, **720 Hz**, 810 Hz, **900 Hz** Hz, 990 Hz, and **1,080 Hz**.

(d) A Just seventh above 60 Hz equals 60 Hz times $\frac{15}{8}$ i.e. 112.5 Hz. Harmonics: 112.5 Hz, 225 Hz, 337.5 Hz, 450 Hz, 562.5 Hz, 675 Hz, 787.5 Hz, **900 Hz**, 1,012.5 Hz and 1,125 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant are the Just fourth and Just third with four and three harmonics coinciding respectively. The least consonant is the Just seventh with only one harmonic coinciding.

4. A Cristofori piano has an action with three levers for each of its keys which cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The upwards velocity of the hammer is due to the movement of the downwards key multiplied by the first, second and third levers by factors of 1, 2.4 and 6 times respectively. Calculate the upwards velocity of the hammer when the downwards velocity of the corresponding key is 3 cm per second. After the

second lever of the action has been replaced with a new lever which has a multiplication factor different from 2.4 times, a downwards velocity of the key of 3.6 cm per second is required to give the same upwards velocity of the hammer as before. What is the multiplication factor of the new second lever?

Answer: Since the three levers when acting together will have a combined multiplication factor equal to 1 times 2.4 times 6 i.e. 14.4, the upwards velocity of the hammer is equal to 3 cm per second times 14.4 i.e. 43.2 cm per second. After a new second lever has been installed, a downwards key velocity of 3.6 cm per second is required for the same upwards velocity of the hammer. The new combined multiplication factor for the three levers acting together is now equal to 43.2 cm per second divided by 3.6 cm per second i.e. 12. Therefore the multiplication factor of the new second lever is equal to 12 divided by 6, i.e. 2 times for the new second lever.

5. On a particular grand piano, the soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal on a particular grand piano are functioning as normal. When a pianist plays on this piano, each of the following four different situations regarding the pedals may occur. In situation 1, after the sostenuto pedal is depressed, the keys D3 and A3 are depressed. The keys D3 and A3 are then released and the sostenuto pedal kept depressed. Will the D3 and A3 notes be sustained in this case? In situation 2, the notes D3 and A3 are depressed, then the

sostenuto pedal is depressed. The D3 and A3 keys are then released while the sostenuto pedal remains depressed. Will the notes D3 and A3 be sustained? In situation 3, after the notes D3 and A3 are depressed, the soft pedal is depressed, and then these keys are released. If the pianist keeps depressing the soft pedal will the notes D3 and A3 be sustained? In situation 4, the sustain pedal is depressed, then the keys D3 and A3 are depressed and then released. If the sustain pedal remains depressed, will the D3 and A3 notes remain sustained?

Answer: If the sostenuto pedal is depressed after the keys D3 and A3 have been depressed, the notes D3 and A3 will be sustained even after their keys have been released, as long as the sostenuto pedal is kept depressed. But if the sostenuto pedal is depressed before the D3 and A3 keys have been depressed, this will not happen. The notes will therefore be sustained only in situation 2 but not in situation 1. If the sustain pedal is depressed before or while the keys D3 and A3 are depressed, the notes D3 and A3 will be sustained as long as the sustain pedal is depressed, and therefore the notes D3 and A3 will be sustained in situation 4. The soft pedal has no effect on the sustaining of the notes, so in situation 3 the notes D3 and A3 will not be sustained.

Scientific Inquiry discussion points

(a) It has now been ascertained, by making scientific observations and from the technical knowledge of how the piano action works, that a pianist play-

ing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontrovertible proof of the earth's roundness. A more recent and current example is climate change. While the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due to man-made causes has become strong enough to be accepted by the great

majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.