PC1221 Fundamentals of Physics I

Lectures 3 and 4

Motion in One Dimension

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Ground Rules

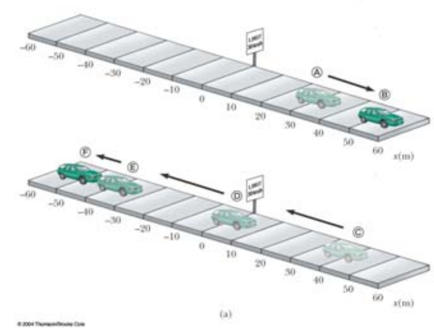
- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

Kinematics

- Describes motion while ignoring the agents that caused the motion
- In these lectures we will consider motion in one dimension (along a straight line)
- We will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size (i.e., the size is very very small)

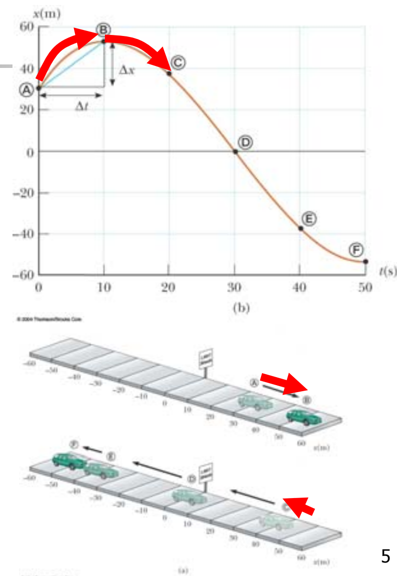
Position

- Position is defined in terms of a **frame of reference**
  - One dimensional, so generally the x-axis or y-axis
- The object's position is its location with respect to the frame of reference
- The frame can be stationary or moving



Position-Time Graph

- The position-time graph shows the motion of the particle (car)
- The smooth curve is an approximation as to what happened between the points



Displacement

- Defined as the change in position during some time interval
  - Represented as Δx (pronounced as delta x)

    $\Delta x = x_f - x_i$ (final position – initial position)
  - SI units are meters (m), Δx can be positive or negative
- Displacement may not be always be equal to distance. Distance refers to the length of a path followed by a particle. Give an example of distance of 5 m but the corresponding displacement is 0.

Vectors and Scalars

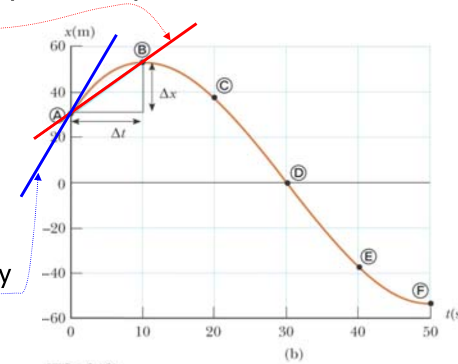
- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - We use **+** and **–** signs to indicate vector directions
- Scalar quantities are completely described by magnitude only

Average Velocity

- The **average velocity** is the rate (w.r.t. time) at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The dimensions for velocity are length / time, [L/T]
- The SI units are m/s, or ms^{-1}
- When Δt becomes a very small time interval, the average velocity is also the instantaneous velocity and its magnitude is the slope of the line (gradient of the tangent line) in the position – time graph



Instantaneous Velocity

- The limit of the average velocity refers to that instance when the time interval becomes infinitesimally short (very very short), or when the time interval approaches zero
- The instantaneous velocity indicates what is happening at every point of time. The information includes the direction of movement and magnitude of velocity.

Average Speed

- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time
- The average speed is not necessarily the magnitude of the average velocity
- Give an example where average speed is 2 m/s and its corresponding velocity is 0.

Instantaneous Velocity, equations

- The general equation for instantaneous velocity is

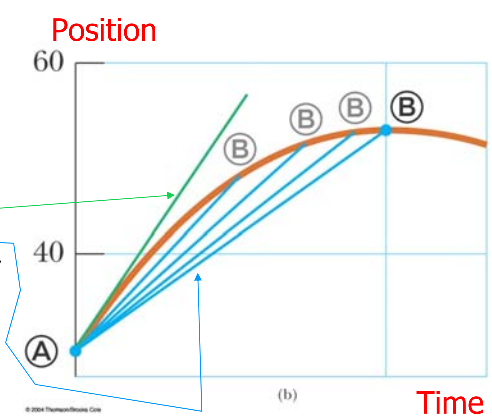
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

(the value of the limit when Δt tends to 0)

- The instantaneous velocity can be positive, negative, or zero

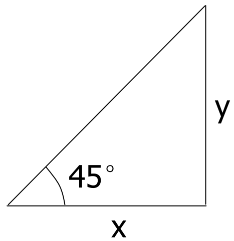
Instantaneous Velocity, graph

- The instantaneous velocity is the slope of the line tangent to the x vs. t curve
- This would be the green line
- The blue lines show that as Δt gets smaller, they approach the green line

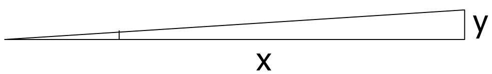


Some Tangent Values

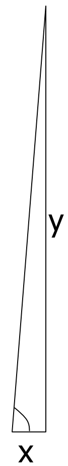
- tan (45°) = y/x = 1



- tan (0°) = 0



- tan (90°) = ∞



Average Acceleration

- Acceleration is the rate of change of the velocity

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$$

- Dimensions are $\frac{L/T}{T}$ = L/T²
- SI units are m/s²

Instantaneous Acceleration

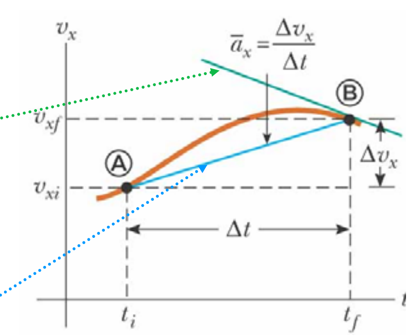
- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0, which is the value of the constant when Δt tends to 0.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

Differentiate ***x*** with respect to ***t*** two times. Please take note that this has nothing to do with square such as $7^2 = 49$.

Instantaneous Acceleration - graph

- The slope of the tangent in the velocity vs. time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration



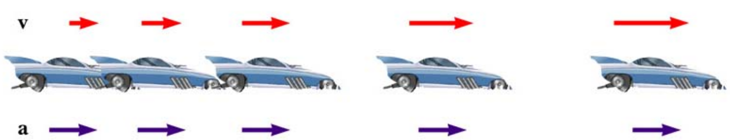
Acceleration and Velocity, 1

- When an object's velocity and acceleration are in the same direction, the object is speeding up
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down

Acceleration and Velocity, 2

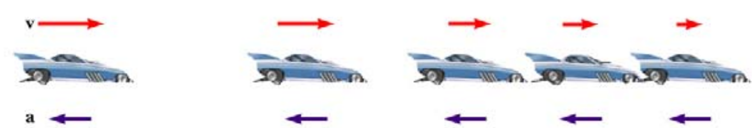
- The car is moving with constant positive velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Acceleration and Velocity, 3



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 4



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

Kinematic Equations - summary

Table 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

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Similar set of equations:

$v = u + at$

$s = s_0 + ½\,(u + v)t$

$s = s_0 + ut + ½\,at^2$

$v^2 = u^2 + 2a\,(s - s_0)$

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ x_f &= x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\,t \\ x_f &= x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \\ v_{xf}^2 &= v_{xi}^2 + 2a_x(x_f - x_i) \end{aligned}$$

u : initial velocity v : final velocity

s_0 : initial displacement s : final displacement

a : acceleration

We will derive them!!

Linear Acceleration

Let Final velocity be v
Initial velocity be u
Distance travelled be s
Acceleration be a
Time duration be t

By definition: $a = \dfrac{v-u}{t}$

$at = v - u$

$\therefore v = u + at$

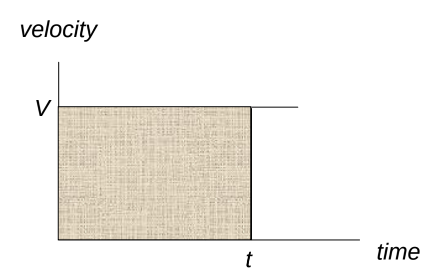
Let the small change in velocity be Δv $\Rightarrow \Delta v = at$

$\therefore v = u + \Delta v$

i.e. final velocity = initial velocity + small change in velocity in the time interval t

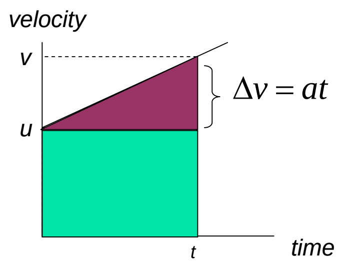
Consider a constant velocity: $v = \dfrac{s}{t} \Rightarrow s = vt$

In v vs. t graph, $v \times t =$ Area under the curve



i.e. the distance travelled is equal to the area under the curve of v vs. t graph.

Now consider a linear change in velocity in the following graph:



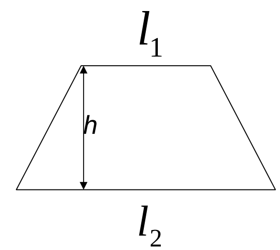
Area under the curve = area of rectangle + area of triangle

$$= ut + \frac{1}{2}(t)(at)$$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

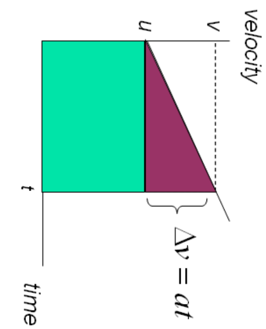
If the starting distance is not 0, say s_0, we have

$$s = s_0 + ut + \frac{1}{2}at^2$$



The distance travelled due to linear acceleration can also be derived by the area of a trapezium:

$$Area = \frac{(l_1 + l_2)}{2} \times h$$



$$\Rightarrow s = \frac{(u+v)}{2} \times t$$

Next, from $v = u + at$, we have $t = \frac{v-u}{a}$

Substitute $\frac{v-u}{a}$ into $s - s_0 = ut + \frac{1}{2}at^2$

We have:

$$s - s_0 = \frac{u(v-u)}{a} + \frac{a(v-u)^2}{2a^2}$$

$$s - s_0 = \frac{uv - u^2}{a} + \frac{a(v^2 - 2vu + u^2)}{2a^2}$$

$$s - s_0 = \frac{2uv - 2u^2}{2a} + \frac{v^2 - 2vu + u^2}{2a}$$

$$s - s_0 = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow 2a(s - s_0) = v^2 - u^2$$

$$\boxed{v^2 = u^2 + 2a(s - s_0)}$$

If the starting point is at origin, i.e. $s_0 = 0$

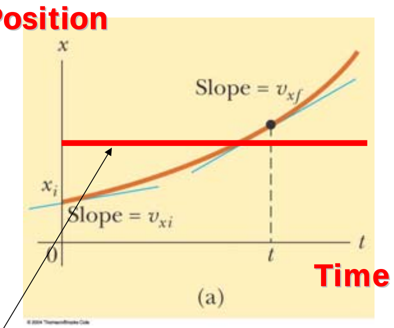
$$\Rightarrow v^2 = u^2 + 2as$$

Kinematic Equations

- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration
- Sometime you may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

Graphical Look at Motion – **Position – time** curve (graph)

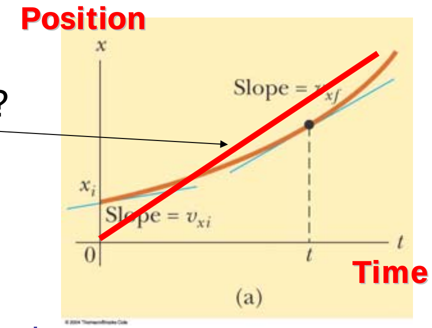
- The slope of the curve is the velocity
- The curved line indicates that the velocity is changing
  - Therefore, there is an acceleration



What if the curve in the Position-Time graph is a horizontal straight line?

Answer: The object is not moving, i.e., velocity = 0.

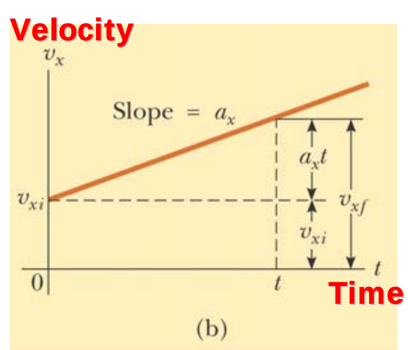
- What if the straight line is not horizontal?



Answer: The object is moving with constant velocity, i.e., no acceleration.

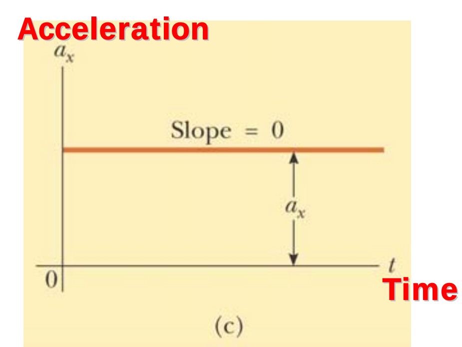
Graphical Look at Motion – **velocity – time** curve

- The slope gives the acceleration
- The straight line in velocity – time graph indicates a constant acceleration, which can be 0 if it is an horizontal line.

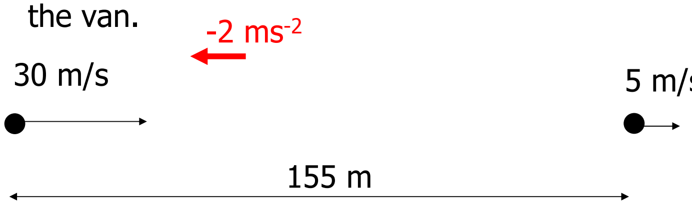


Graphical Look at Motion – **acceleration – time** curve

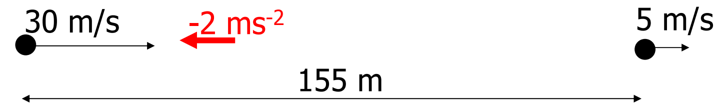
- The zero slope in acceleration – time graph indicates a constant acceleration



Example: Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at –2.00 m/s^2 because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.



Answer:



Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = \cancelto{0}{x_{is}} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}(-2.00\ \text{m/s}^2)t^2 .$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0 .$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$
$$0 = t_c^2 - 25.0t_c + 155 .$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}} .$$

Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object, which can be:
  - Dropped – released from rest
  - Thrown downward
  - Thrown upward

Acceleration of Freely Falling Object

- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s^2
  - g varies slightly at different geographical locations
  - 9.80 m/s^2 is the average at the Earth's surface

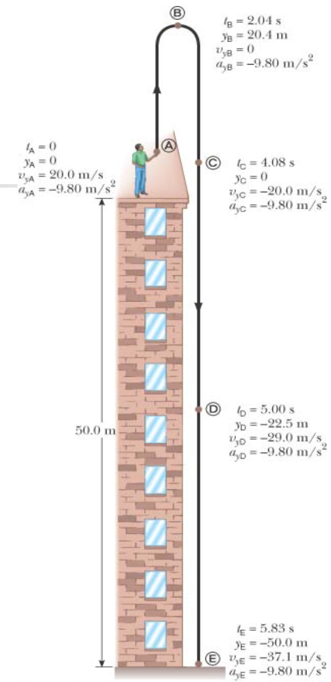
Acceleration of Free Fall, cont.

- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive, so downward is negative
- Use the kinematic equations with

  $a_y = g = -9.80\ \text{m/s}^2$

Free Fall Example

- Initial velocity at A is upward (+) and acceleration is g (-9.8 m/s^2)
- At B, the velocity is 0 and the acceleration is g (-9.8 m/s^2)
- At C, the velocity has the same magnitude as at A, but is in the opposite direction
- At E, the displacement is –50.0 m (it ends up 50.0 m below its starting point)



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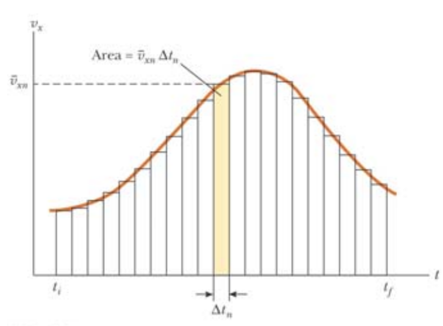
Motion Equations from Calculus

- Displacement equals the area under the velocity – time curve

  $$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

- The limit of the sum is a definite integral



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x\,dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x\,dt$$

Kinematic Equations – Calculus Form with Constant Acceleration

- The integration form of $v_f - v_i$ gives

$$v_{xf} - v_{xi} = a_x t$$

$v = u + at$, or

$v - u = at$

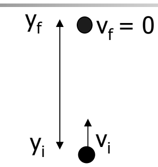
- The integration form of $x_f - x_i$ gives

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$s = s_0 + ut + ½\, at^2$, or

$s - s_0 = ut + ½\, at^2$

Example: A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the height it reaches.



Answer:

(a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00\ \mathrm{s}$, $g = 9.80\ \mathrm{m/s^2}$. Therefore,

$$v_i = gt = \left(9.80\ \mathrm{m/s^2}\right)\left(3.00\ \mathrm{s}\right) = \boxed{29.4\ \mathrm{m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}\left(v_f + v_i\right)t$

$$y_f - y_i = \frac{1}{2}\left(29.4\ \mathrm{m/s}\right)\left(3.00\ \mathrm{s}\right) = \boxed{44.1\ \mathrm{m}}$$

Example: Two stones are released from rest at a certain height, one after the other at an interval of *c* seconds.

(a) Will the difference in their speeds increase, decrease, or stay the same?

Answer:

Let $v1$ be the speed of first stone, and $v2$ the speed of second stone. We have

$v1 = 0 - gt$

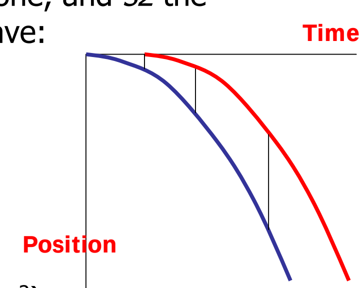
$v2 = 0 - g(t-c)$;

$v1 - v2 = -gt + g(t-c) = -gc$ **constant w.r.t. time**

(b) Will their separation distance increase, decrease or stay the same?

Answer:

Let $s1$ be the displacement of first stone, and $s2$ the displacement of second stone. We have:



$s1 = 0 - ½\, gt^2$ for $t \geq 0$

$s2 = 0 - ½\, g(t - c)^2$ for $t \geq c$

$s1 - s2 = -½\, gt^2 + ½\, g(t - c)^2$

$= -½\, gt^2 + ½\, g(t^2 - 2tc + c^2)$

$= ½\, g(-2tc + c^2)$ **absolute difference is increasing w.r.t. time**

(c) Will the time interval between the instants at which they hit the ground be smaller than, equal to, or larger than the time interval between the instants of their release?

Answer:

Let *t1* be the time for the first stone to hit the ground, and *t2* the time for the second stone to hit the ground. Let the height be *h*. We have:

$$-h = 0 - \tfrac{1}{2} g\, t1^2$$

$$t1^2 = \frac{2h}{g}$$

$$t1 = \sqrt{\frac{2h}{g}}$$

As the distance travelled by the second stone is the same. We have

$$t2 = c + \sqrt{\frac{2h}{g}}$$

Therefore $t2 - t1 = c$ (same as the difference of the instants of their release.)

Example: A small piece of light paper and a metal coin are dropped from the same height in the air at the same time. Which item will reach the floor first?


Answer:

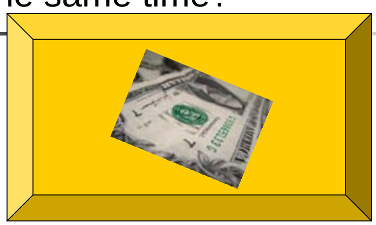
The metal coin will reach the ground first.

Is the acceleration due to gravity experienced by the small piece of paper is smaller (as it takes a longer time to reach the ground) than that experienced by the metal coin? Why do the two objects reach the floor at different time?

Answer:

The acceleration due to gravity is the same for both items. But the effect of upthrust is more significant on the paper due to its light weight and wide area. The net acceleration is therefore not he same on both objects thus their different arrival times.

Example: What if the paper is on top of a metal plate and all the 3 items are released at the same time?




Answer:

The metal plate has removed the air volume below the paper during the free fall so the paper and the coin will reach the ground at the same time. There is almost no upthrust due to the air on the paper.

How to make the paper and coin hit the floor at the same time without the help of metal plate, container and air pump?


Answer:

There is nothing you can do with the weight of the paper. But you can do something about its area. How?