

PC1221 Fundamentals of Physics I

Lectures 11 and 12

Circular Motion
and
Other Applications of Newton's Laws

A/Prof Tay Seng Chuan

1

Ground Rules

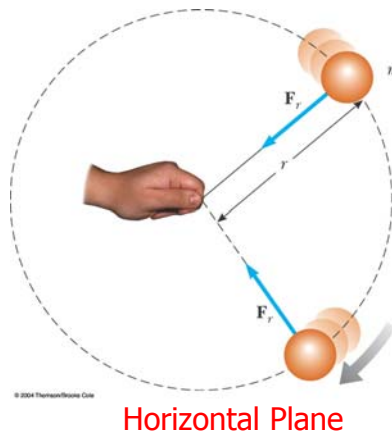
- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecture notes to lecture

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Uniform Circular Motion (on Horizontal Plane)

- A force, F_r , is directed toward the center of the circle
- This force is associated with an acceleration, a_c
- Applying Newton's Second Law along the radial direction gives

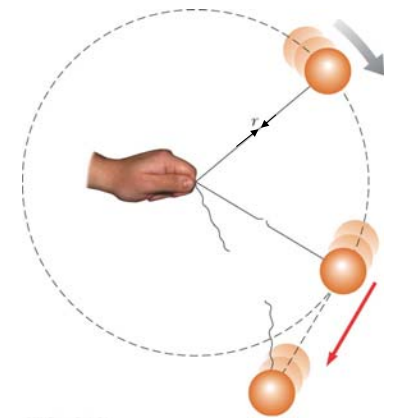
$$\sum F = ma_c = m \frac{v^2}{r}$$



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Uniform Circular Motion, cont

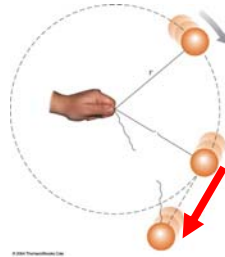
- A force (from my hand) causes a centripetal acceleration to act toward the center of the circle
- It causes a change in the direction of the velocity vector
- If the force vanishes, the object would move in a straight-line path tangent to the circle



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Centripetal Force

- The force causing the centripetal acceleration is called the *centripetal force*
- This is not a new force, but it is a new *role* for a force
- Centripetal force causes circular motion**



Centripetal Force

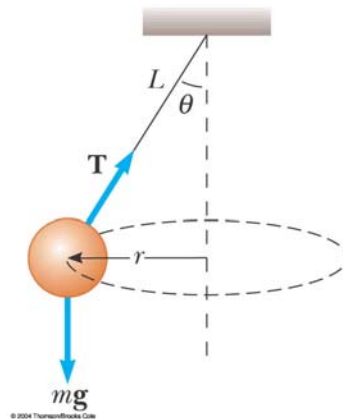


Centripetal force causes circular motion. When centripetal force is not applied, the object continues with linear motion tangential to the curve of the circular motion.

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Conical Pendulum

- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction
- $v = \sqrt{Lg \sin \theta \tan \theta}$
- v is independent of m
- This formula can be derived



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Conical Pendulum

$$\text{Centripetal force} = ma = \frac{mv^2}{r} = T \sin \theta \quad \text{-- (1)}$$

$$T \cos \theta = mg$$

$$\Rightarrow T = \frac{mg}{\cos \theta} \quad \text{-- (2)}$$

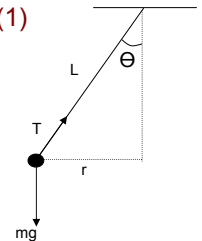
Sub (2) into (1):

$$\frac{mv^2}{r} = \frac{mg}{\cos \theta} \sin \theta$$

$$\Rightarrow \frac{v^2}{r} = g \tan \theta \quad \text{-- (3)}$$

$$\text{But } \sin \theta = \frac{r}{L}$$

$$\Rightarrow r = L \sin \theta \quad \text{-- (4)}$$



Sub (4) into (3):

$$\frac{v^2}{L \sin \theta} = g \tan \theta$$

$$\Rightarrow v^2 = Lg \sin \theta \tan \theta$$

$$v = \sqrt{Lg \sin \theta \tan \theta}$$

Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord
- The centripetal force is supplied by the tension

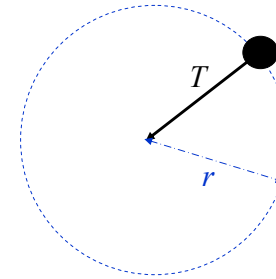
$$v = \sqrt{\frac{Tr}{m}}$$

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Motion in a Horizontal Circle

Tension (T) on the string:

$$T = \frac{mv^2}{r} \Rightarrow T \times r = mv^2$$

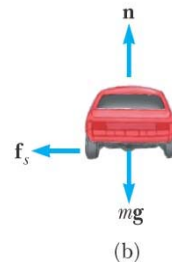
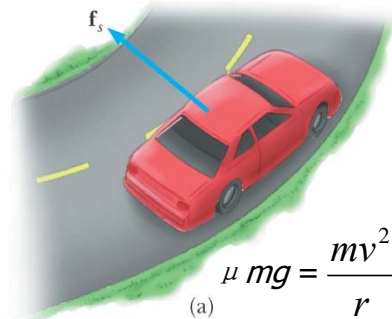


$$v^2 = \frac{T \times r}{m}$$

$$\Rightarrow v = \sqrt{\frac{T \times r}{m}}$$

Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
- The maximum speed at which the car can negotiate the curve is $v = \sqrt{\mu gr}$
- Note, the maximum velocity does not depend on the mass of the car



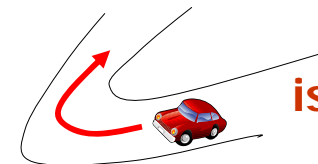
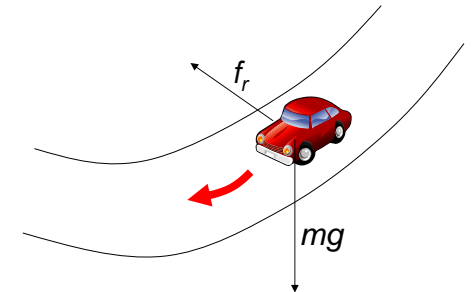
Maximum Velocity on a Flat Race Track

f_r : friction

$$f_r = \mu mg = \frac{mv^2}{r}$$

$$\Rightarrow \mu gr = v^2$$

$$\text{So } v = \sqrt{\mu gr}$$



What if this is a sharp turn?

Example. A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.6, how fast can the truck be moving without the crate sliding?

Answer:

$$n = mg \text{ since } a_y = 0$$

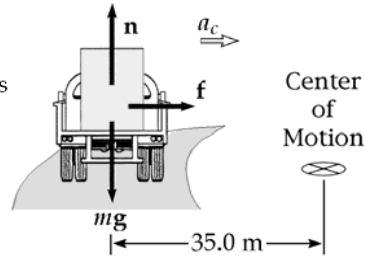
The force causing the centripetal acceleration is the frictional force f .

$$\text{From Newton's second law } f = m a_c = \frac{m v^2}{r}.$$

$$\text{But the friction condition is } f \leq \mu_s n \text{ i.e., } \frac{m v^2}{r} \leq \mu_s m g$$

$$v \leq \sqrt{\mu_s r g} = \sqrt{0.600(35.0 \text{ m})(9.80 \text{ m/s}^2)}$$

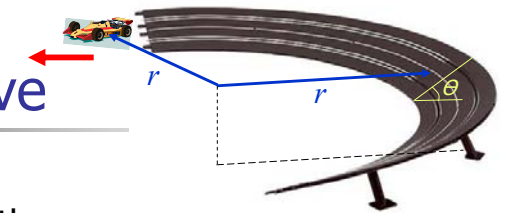
$$v \leq \boxed{14.3 \text{ m/s}}$$



What if $v > 14.3 \text{ m/s}$?

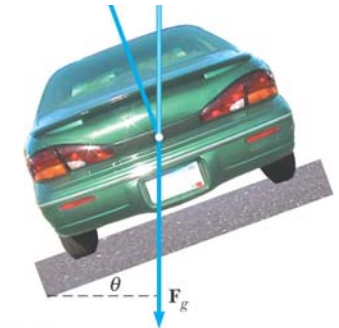
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Banked Curve



- Banked curves are designed to deal with very small or no friction situation
- There is a component of the normal force that supplies the centripetal force

$$\tan \theta = \frac{v^2}{r g}$$



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Speed on a banked Curve

$$\text{Vertically: } n \cos \theta = mg$$

$$\Rightarrow n = \frac{mg}{\cos \theta} \quad - (1)$$

$$\text{Horizontally: } n \sin \theta \text{ is the centripetal force}$$

$$n \sin \theta = \frac{m v^2}{r} \quad - (2)$$

$$\text{Sub (1) into (2): } \frac{mg}{\cos \theta} \sin \theta = \frac{m v^2}{r}$$

$$g \tan \theta = \frac{v^2}{r}$$

$$\Rightarrow \tan \theta = \frac{v^2}{r g}$$

$$\text{So } v = \sqrt{r g \tan \theta}$$



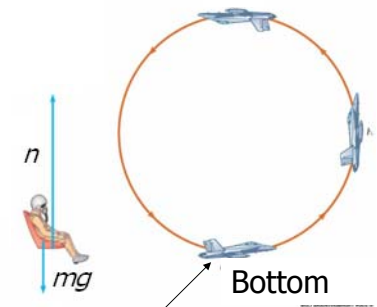
What if $v < \sqrt{r g \tan \theta}$?

What is $v > \sqrt{r g \tan \theta}$?

Loop-the-Loop in Vertical Plan

- This is an example of a vertical circle
- At the bottom of the loop, the upward force experienced by the object is greater than its weight

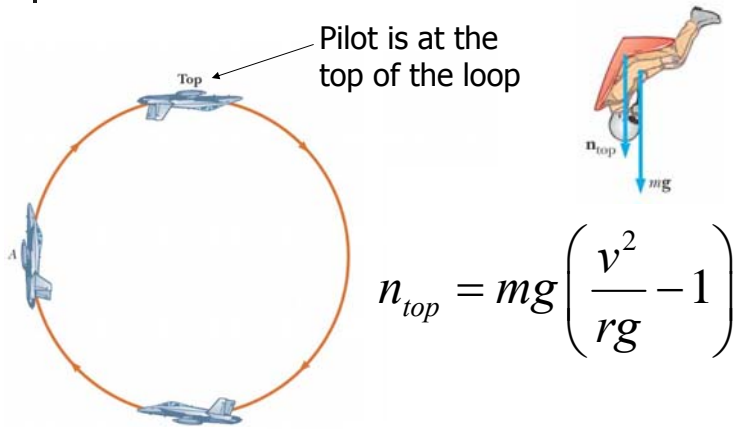
$$n_{bot} = mg \left(1 + \frac{v^2}{r g} \right)$$



Pilot is at the bottom of the loop

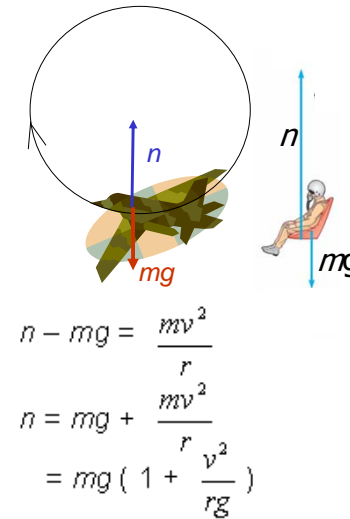
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Loop-the-Loop, Part 2

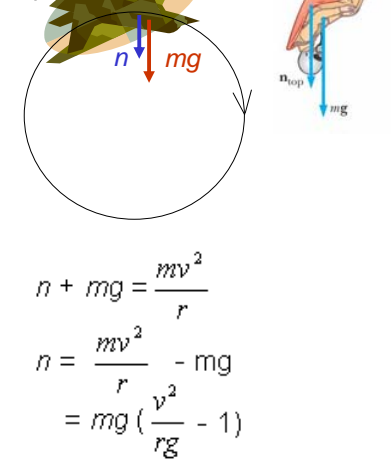


Loop-the-loop in vertical plan

(1) Bottom

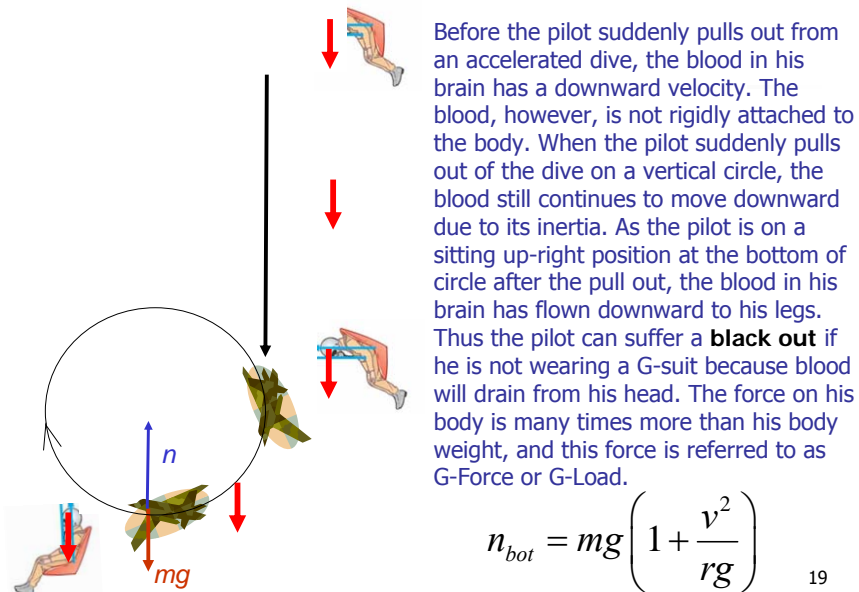


(2) Top

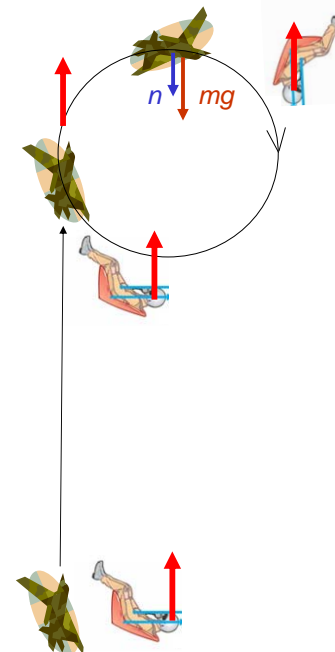


How can a pilot become black out during a flight?

Answer:

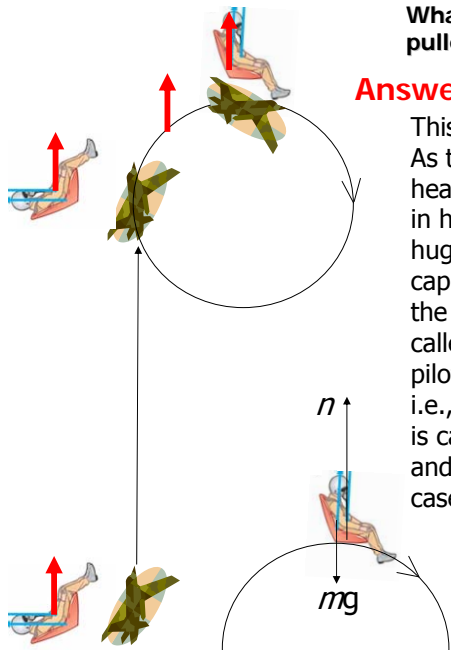


Another situation of black out



Before the pilot suddenly pulls out from an accelerated climb in a circle as shown in the diagram in an up-side-down position, the blood in his brain has a upward velocity. When the pilot suddenly pulls out of the climb on a vertical circle, his blood still continues to move upward due to its inertia. As the pilot is on an up-side-down position at the top of circle after the pull out from the climb, the blood in his brain has flown upward to his legs. Thus the pilot can also suffer a **black out** during a sudden pull out of an accelerated climb. The G-force for this case is **usually positive** due to the very high velocity of the aircraft and the sudden pull out from the climb, i.e., $v \uparrow$, and $r \downarrow$.

$$n_{top} = mg \left(\frac{v^2}{rg} - 1 \right)$$



What if the accelerated climb is suddenly pulled out in an up-right position?

Answer:

This is called the **red out** situation. As the blood suddenly floods to the head of the pilot, the small capillaries in his eyes will be flooded with a huge flow of blood. When the capillaries in eyes start to rupture, the blood will blur his vision – thus called **red out**. The G-force on the pilot in this case is **usually negative**, i.e., n can be negative if the pull out is carried out at **very high velocity** and sudden from the climb in this case.

$$mg - n = \frac{mv^2}{r}$$

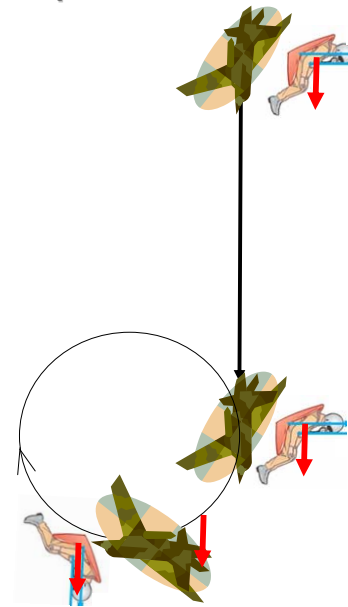
$$n = mg - \frac{mv^2}{r}$$

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Why if the sudden pull out from the dive is up-side-down?

Answer:

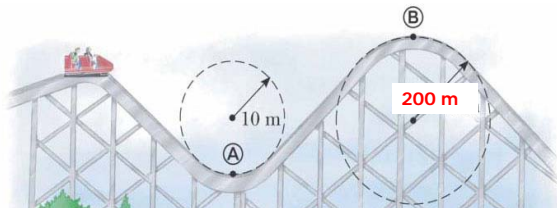
This will be a **red-out** situation during a sudden pull out from the dive if the pilot is up-side down. The G-force on the pilot is negative, i.e., n is negative.



$$-n - mg = \frac{mv^2}{r}$$

$$n = -\frac{mv^2}{r} - mg$$

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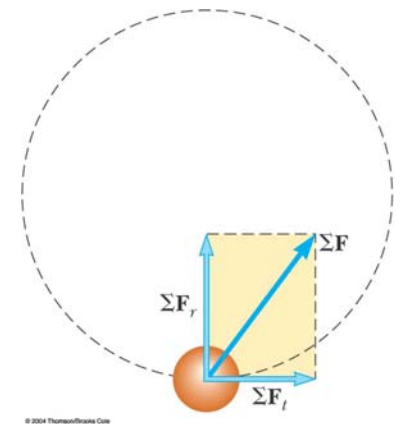
Your stomach is largely filled with porridge when you go to the theme park. When you sit on a roller coaster car that travels smoothly initially, and later accelerates from point A onwards for more than 200m uphill, you feel like vomiting. Which is the most likely position where you feel like vomiting (before A, A, B, or after B)?

Answer:

In actual case you can feel like vomiting every where if your balancing system is not used to the speed and rotation, even before you go to the them park or when you have left the theme park. But the most vulnerable position is at B.

Non-Uniform Circular Motion

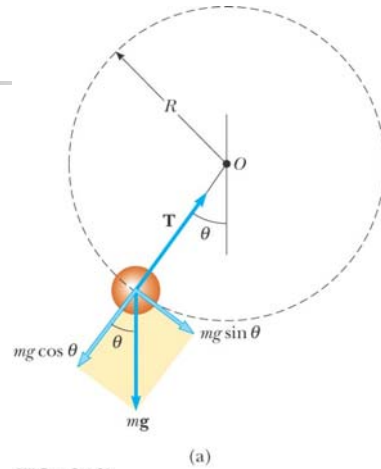
- The acceleration and force have tangential components
- F_r produces the centripetal acceleration
- F_t produces the tangential acceleration
- $\Sigma F = \Sigma F_r + \Sigma F_t$



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Vertical Circle with Non-Uniform Speed

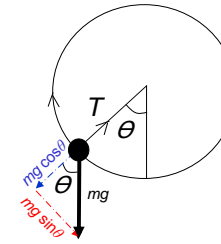
- The gravitational force exerts a tangential force on the object
 - Look at the components of F_g
- The tension at any point can be found



$$T = m \left(\frac{v^2}{R} + g \cos \theta \right)$$

This formula corresponds to the previous 2 special cases when $\theta = 0^\circ$ and 180° .

Vertical Circle with Non-uniform Speed



$$T - mg \cos \theta = \frac{mv^2}{r}$$

$$\Rightarrow T = \frac{mv^2}{r} + mg \cos \theta$$

$$= mg \left(\frac{v^2}{rg} + \cos \theta \right)$$

$$= m \left(\frac{v^2}{r} + g \cos \theta \right)$$

When $\theta = 0^\circ$ (Bottom), $T = mg \left(\frac{v^2}{rg} + 1 \right)$

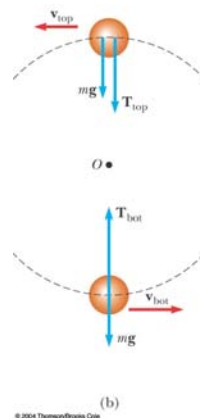
When $\theta = 180^\circ$ (Top), $T = mg \left(\frac{v^2}{rg} - 1 \right)$

Top and Bottom of Circle

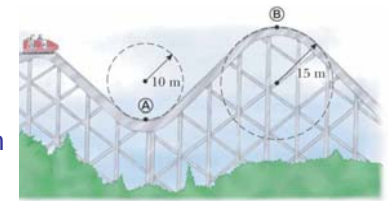
- The tension at the bottom is a maximum
- The tension at the top is a minimum
- If $T_{top} = 0$, then

$$v_{top} = \sqrt{gR}$$

What if the velocity is less than this value?



Examples. A roller coaster car has a mass of 500 kg when fully loaded with passengers. (a) If the vehicle has a speed of 20.0 m/s at point A, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the vehicle can have at B and still remain on the track?



Answer

(a) $v = 20.0 \text{ m/s}$,

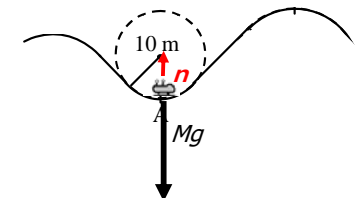
n = force of track on roller coaster, and

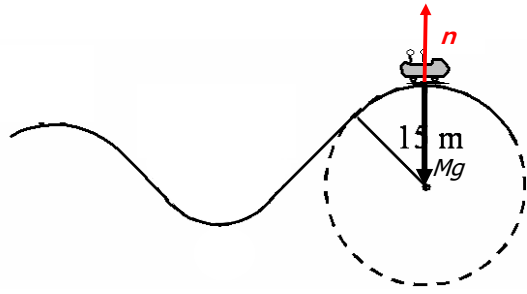
$R = 10.0 \text{ m}$.

$$\sum F = \frac{M v^2}{R} = \underline{n - Mg}$$

$$n = Mg + \frac{M v^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)^2}{10.0 \text{ m}}$$

$$n = 4900 \text{ N} + 20000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$





(b) At B, $Mg - n = \frac{M v^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$Mg = \frac{M v_{\text{max}}^2}{R} \Rightarrow v_{\text{max}} = \sqrt{Rg} = \sqrt{15.0(9.80)} = 12.1 \text{ m/s}$$

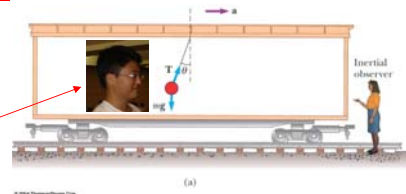
Have you entered a ghost house in Disney Land?



Motion in Accelerated Frames

- A ***fictitious force*** results from an accelerated frame of reference
 - A fictitious force appears to act on an (first) object in the same way as a real force, but you cannot identify a second object that applies the fictitious force to the first object. **Therefore a fictitious force is not a real force that exhibits real effects.**

Observer in accelerated frame (non-inertial)



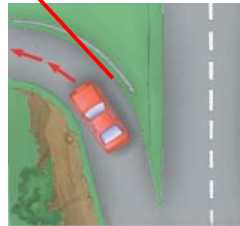
"Centrifugal" Force

- From the frame of the passenger (b), a force appears to push her toward the door
- From the frame of the Earth, the car applies a leftward force on the passenger. This is the centripetal force
- The outward force is often called a ***centrifugal*** force
 - It is a fictitious force due to the acceleration associated with the car's change in direction





What if the car skid at the corner?



Answer:

The car will lose traction on the road surface and will slide along the tangent of the curvature where it skids.



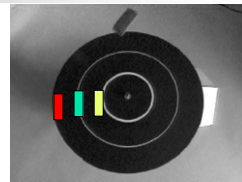
Why did the rider bend toward the center of curvature when turning a corner?

Answer:

To counter act the fictitious force on him so that he can maintain the balance.

Which eraser will fly off first?

Three erasers are placed on a rotating disc at different distances from center and we spin the disc at increasing speed. Which eraser will fly off first (inner, middle, outer)? Why?

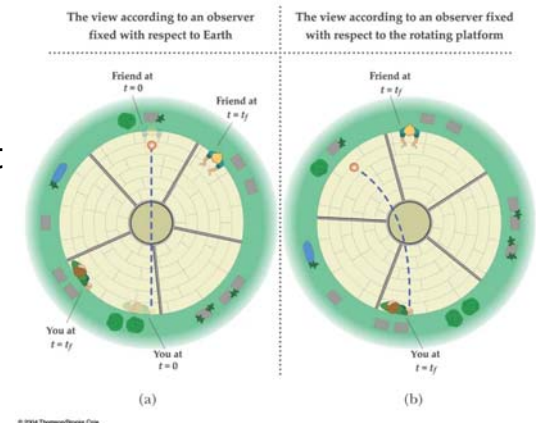


Answer:

All three erasers experience the same friction on the disc surface. The centripetal force on each eraser is different: $m v^2/r = m(r\omega)^2/r = mr\omega^2$, where ω is the angular velocity. As the **outer eraser** (with the largest r) experiences the largest centripetal force, its **fictitious** outward force (**centrifugal force**) is the largest so it will fly off first.

“Coriolis Force”

- This is an apparent force caused by changing the radial position of an object in a rotating coordinate system
- The result of the rotation is the curved path of the ball

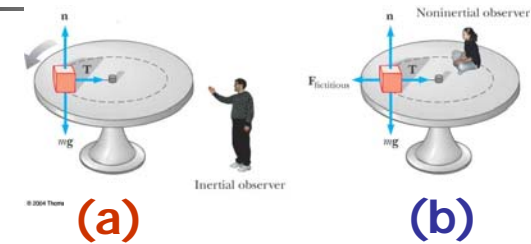


Fictitious Forces, examples

- Although fictitious forces are not real forces, they can have real effects
- Examples:
 - Objects in the car do slide
 - You can feel pushed to the outside of a rotating platform
 - The Coriolis force is responsible for the rotation of weather systems and ocean currents.

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Fictitious Forces in a Rotating System



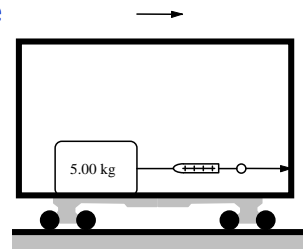
- According to the inertial **observer (a)**, the tension is the centripetal force $T = \frac{mv^2}{r}$

- The noninertial **observer (b)** sees

$$T - F_{\text{fictitious}} = T - \frac{mv^2}{r} = 0$$

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Example. An object of mass 5.00 kg, attached to a spring scale, rests on a frictionless, horizontal surface as in Figure. The spring scale, attached to the front end of a boxcar, has a constant reading of 18.0 N when the car is in motion. (a) If the spring scale read zero when the car was at rest, determine the acceleration of the car. (b) What constant reading will the spring scale show if the car moves with constant velocity? (c) Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.



Answer:

(a) $\sum F_x = M a, a = \frac{T}{M} = \frac{18.0 \text{ N}}{5.00 \text{ kg}} = \boxed{3.60 \text{ m/s}^2}$ to the right.

(b) If $v = \text{const}$, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are (i) T and (ii) a fictitious force $(-Ma)$. Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x -direction.