PC1221 Fundamentals of Physics I

Lectures 13 and 14

Energy and Energy Transfer

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Ground Rules

- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

Introduction to Energy

- The concept of energy is one of the most important topics in science while it is not the only important topic
- Every physical process that occurs in the Universe involves energy and energy transfers or energy transformations
- Giving a lecture now is an example of the transfer and transform of energy but it is more complicated (chemical energy from the food I ate this morning is used to create the sound energy in my voice now)

Energy Approach to Problems

- The energy approach to describing motion is particularly useful when the force is not constant
- An approach will involve *Conservation of Energy*
  - This could be extended to biological organisms, technological systems and engineering situations

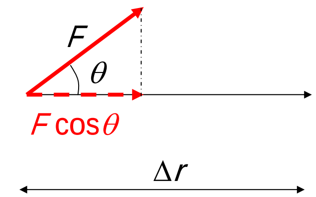
Systems

- A *system* is a small portion of the Universe
- A valid system may
  - be a single object or particle
  - be a collection of objects or particles
  - be a region of space
  - vary in size and shape
- Energy in a system is conserved.

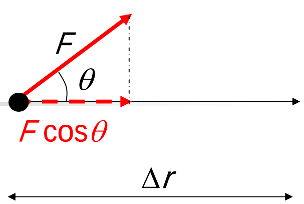
Work

- The work, W, done on a system by an agent exerting a constant force on the system is the product of the magnitude, F, of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and the displacement vectors.

$W = F \Delta r \cos\theta$



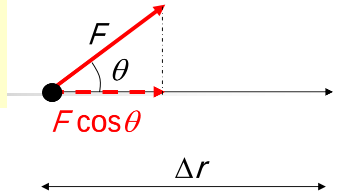
Work, cont.



- $W = F \Delta r \cos\theta$
  - The displacement is that of the point of application of the force
  - A force does no work on the object if the force does not move through a displacement
  - The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application

Did you do any work if you were repelling along a rope down a helicopter? Did the gravity do work on you?

Did you do any work if you were running on a level ground?

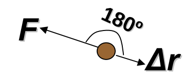


Why did you feel tired after the run?

Example. **In a baseball game, the catcher stops a 90-mph pitch. What can you say about the work done by the catcher on the ball?**

1. catcher has done positive work
2. catcher has done negative work
3. catcher has done zero work

Answer:



The force exerted by the catcher is opposite in direction to the displacement of the ball, so the work is negative. Or using the definition of work ($W = F(\Delta r)\cos\theta$), because $\theta = 180°$, then $W < 0$. Note that because the work done on the ball is negative, its speed decreases.

Example. **A box is being pulled up a rough incline by a rope connected to a pulley. How many forces are doing work on the box?**

Answer:

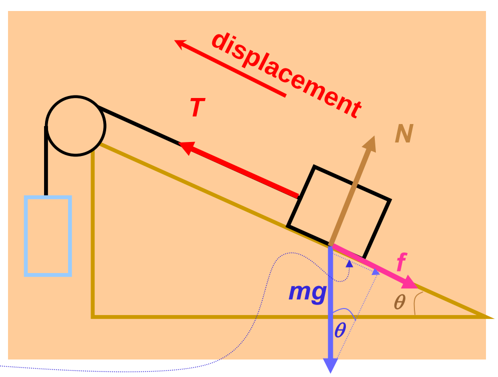
Any force not perpendicular to the motion will do work:

N **does** ***no work***

T **does** **positive work**

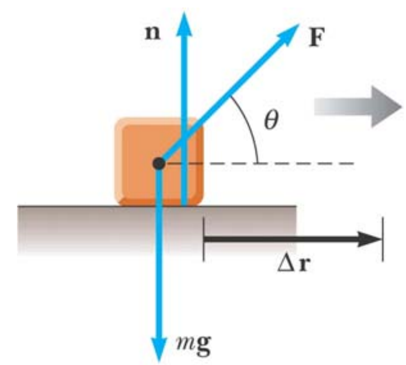
f **does** **negative work**

mg **sin*θ*** **does negative work**



Work in Pushing a Block

- The normal force, n, and the gravitational force, mg, do no work on the object
  - $\cos\theta = \cos 90° = 0$
- The force **F** does do work on the object



More About Work

- The system and the environment must be determined when dealing with work
  - The environment does work on the system
    - Work **by** the environment **on** the system
- The sign of the work depends on the direction of **F** relative to Δ**r**
  - Work is positive when projection of **F** onto Δ**r** is in the same direction as the displacement
  - Work is negative when the projection is in the opposite direction

Units of Work

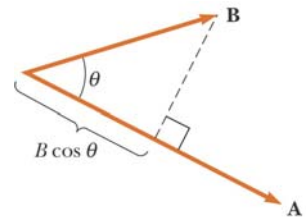
- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter
  - J = N · m

Work Is An Energy Transfer

- If the work is done on ~~(by)~~ a system and it is positive, energy is transferred to the system
- If the work done on ~~(by)~~ the system is negative, energy is transferred from the system
- If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary (e.g., heat generated due to friction)
  - This will result in a change in the amount of energy stored in the system

Scalar Product of Two Vectors

- The scalar product of two vectors is written as **A** · **B** (read as A dot B)
  - It is also called the dot product
- **A** · **B** = $A\,B\cos\theta$
  - θ is the angle *between* A and B



$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

Scalar Product, cont

- The scalar product is commutative
  - **A · B = B · A**
- The scalar product obeys the distributive law of multiplication
- **A · (B + C) = A · B + A · C**

Dot Products of Unit Vectors

- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

  $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$

- Using component form with **A** and **B**:

  $A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

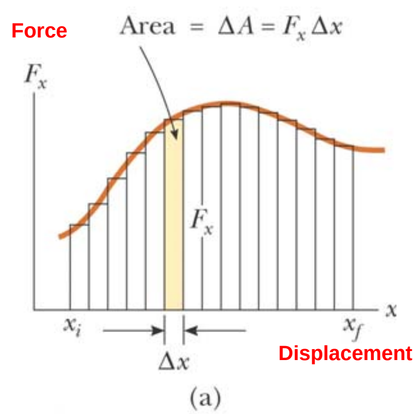
  $B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

  $A \cdot B = A_xB_x + A_yB_y + A_zB_z$

Work Done by a Varying Force

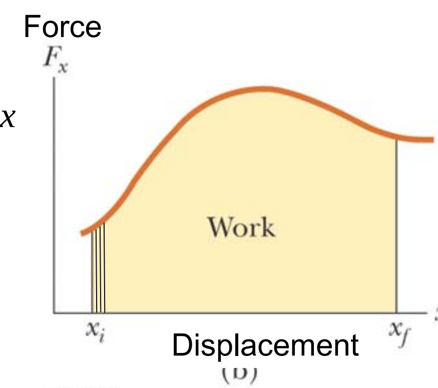
- Assume that during a very small displacement, Δx, F is constant
- For that displacement, $W \sim F\Delta x$
- For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont

- $\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$
- Therefore, $W = \int_{x_i}^{x_f} F_x dx$
- The work done is equal to the area under the curve of force versus displacement



Work Done By Multiple Forces

- If more than one force acts on a system and the system can be modeled as a particle, the total work done on the system is the work done by the net force

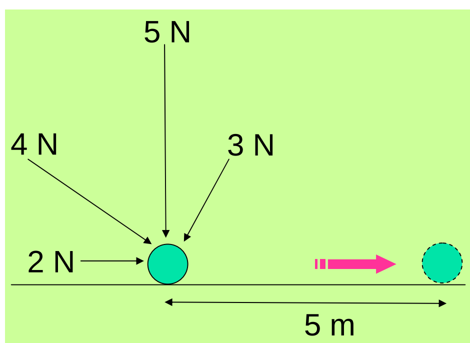
$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

Work Done by Multiple Forces, cont.

- If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

Example. Four forces are acting on an object along a horizontal plane. The force with 2 N is acting along the plane, 4 N with 30º from the horizontal line, 5 N perpendicular to the plane, and 3 N 60º from the horizontal line. What is the net work done of all these forces for the ball to travel a horizontal distance of 5 m shown in the figure?



Answer:

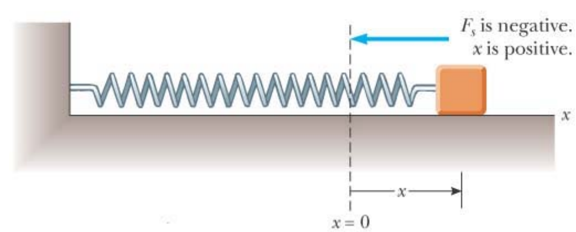
The net horizontal force is 2 + 4 cos 30º - 3 cos 60º = 3.96 N
The net work done = 3.96 N x 5 m = 19.82 J

Or:

The net work done is the sum of the workdone by the individual forces:

2 x 5 + 4 cos 30º x 5 - 3 cos 60º x 5 = 19.82 J

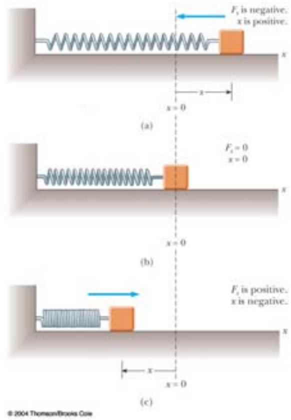
Hooke's Law



- The force exerted by the spring is
  $F_s = - kx$
  - x is the position of the block with respect to the equilibrium position ($x = 0$)
  - k is called the spring constant or force constant and measures the stiffness of the spring
- This is called **Hooke's Law**

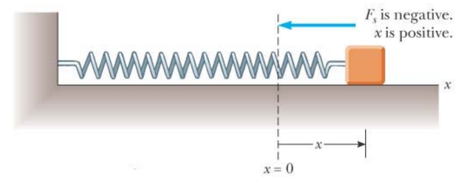
Hooke's Law, cont.

- When *x* is positive (spring is stretched), *F* is negative
- When *x* is 0 (at the equilibrium position), *F* is 0
- When *x* is negative (spring is compressed), *F* is positive



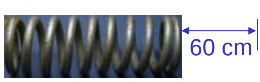
Hooke's Law, final

- The force exerted by the spring is always directed opposite to the displacement from equilibrium
- *F* is called the *restoring force*
- If the block is released it will oscillate back and forth between *x* and –*x*



Spring Combination

A spring can be stretched a distance of 60 cm with an applied force of 1 *N*. If an identical spring is connected in series with the first spring, how much force will be required to stretch this series combination a distance of 60 cm?

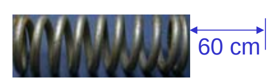


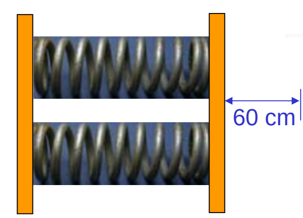
Answer:

Here, the springs are in series, so each spring is only stretched 30 cm thus only half the force is needed. But also, because the springs are in a row, the force applied to one spring is transmitted to the other spring (like tension in a rope). So the overall applied force of 0.5 *N* is all that is needed. The combination of two springs in series behaves like a weaker spring!!

Spring Combination

What if the two springs are arranged in parallel?





Answer:

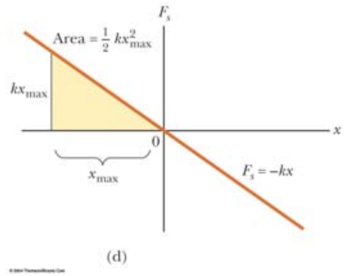
Each spring is still stretched 60 cm, so each spring requires 1 *N* of force. But because there are two springs, there must be a total of 2 *N* of force! Thus, the combination of two parallel springs behaves like a stronger spring!!

Work Done by a Spring

- Identify the block as the system
- Calculate the work as the block moves from
- $x_i = -x_{max}$ to $x_f = 0$, or

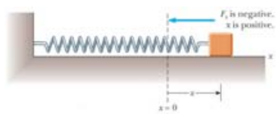
  $x_i = x_{max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_x dx = \int_{-x_{max}}^{0} (-kx)\,dx = \frac{1}{2}kx_{max}^2$$



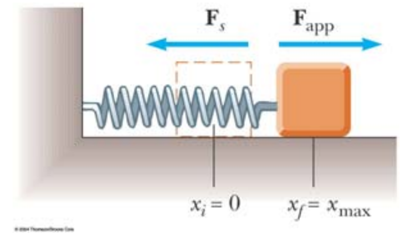
- The total work done as the block moves from

  $-x_{max}$ to x_{max} is zero



Spring with an Applied Force

- Suppose an external agent, F_{app}, stretches the spring
- The applied force is equal and opposite to the spring force
- $F_{app} = -F_s = -(-kx) = kx$
- Work done by F_{app} is equal to ½ kx^2_{max}



Example. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

Answer:

$$4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$$

$$\therefore k = 800\text{ N/m}$$

To stretch the spring to 0.2 m from 0.1 m requires an additional work of

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = \boxed{12.0\text{ J}}$$

Kinetic Energy

- Kinetic Energy is the energy of a particle due to its motion
  - $K = ½\, mv^2$
    - K is the kinetic energy
    - m is the mass of the particle
    - v is the speed of the particle
    - the expression ½ mv^2 can be derived using $F = ma$, and

      $$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
- A change in kinetic energy is one possible result of doing work to transfer energy into a system

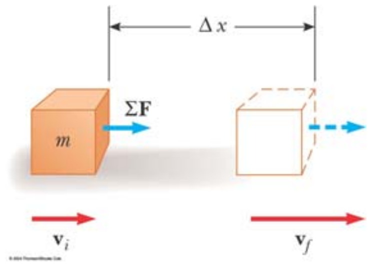
Kinetic Energy, cont

- Calculating the work by integration:

$$W = \int_{x_i}^{x_f} \sum F \, dx = \int_{x_i}^{x_f} ma \, dx$$

$$W = \int_{v_i}^{v_f} mv \, dv$$

$$\sum W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$



$$ma\,dx = m\frac{dv}{dt}dx = mv\,dv$$

Work-Kinetic Energy Theorem

- The Work-Kinetic Energy Principle states $\Sigma W = K_f - K_i = \Delta K$
- In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.
- Kinetic energy possessed by an object of mass *m* and velocity *v* is

$K = ½ \, mv^2$

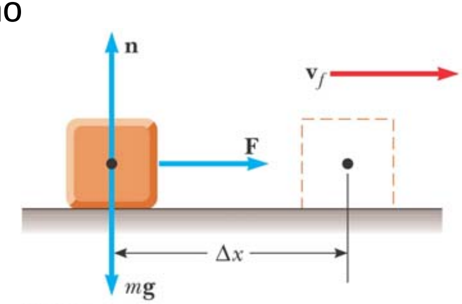
Work-Kinetic Energy Theorem

- The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement

$W = F\Delta x = \Delta K$

$= ½ \, mv^2 - 0$

(the vertical normal force has no effect)

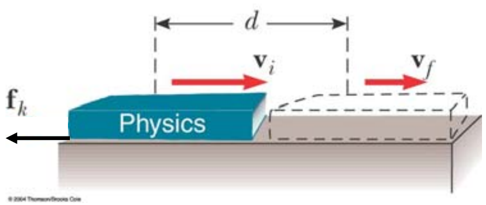


Nonisolated System

- A *nonisolated system* is one that interacts with or is influenced by its environment, e.g., push a block on a rough surface
  - An *isolated system* would not interact with its environment (e.g., push a block on a friction-less surface.)
- The Work-Kinetic Energy Theorem can also be applied to nonisolated systems. In that case there will be a transfer of energy across the boundary of an object (e.g., from the block to the surface where heat is generated.)

Internal Energy

- The energy associated with an object's temperature is called its *internal energy*, E_{int}
- In this example, the surface is the system
- The friction does work and increases the internal energy of the surface

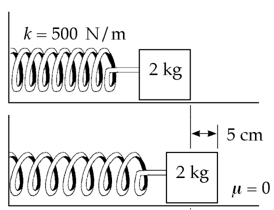


Potential Energy

- *Potential energy* is energy related to the configuration of a system in which the components of the system interact by forces
- Examples include:
  - elastic potential energy – stored in a spring
  - gravitational potential energy, e.g., you stand on top of Bukit Timah Hill
  - electrical potential energy, e.g., you switch on electricity to iron your shirt

Example. A 2.00-kg block is attached to a spring of force constant 500 N/m as shown in figure. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

Answer:

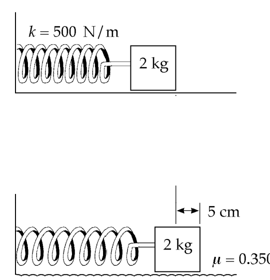


(a) When the block is pulled back to equilibrium position, the potential energy in spring will be fully converted to the kinetic energy in the block when the spring returns to original length, i.e.,

$$\frac{1}{2} k x^2 = \frac{1}{2} m v^2.$$

So, $v = \sqrt{\frac{k}{m}} \times x = \sqrt{\frac{500}{2}} \times 0.05 =$ 0.79 m/s

(b) If there is fiction on the surface, the potential energy on the spring is first wasted on the work done by friction. The remaining energy is converted to the kinetic energy in the block, i.e., $\frac{1}{2} k x^2 - \mu mg x = \frac{1}{2} m v^2$.



In terms of initial sum and final sum of energies, you can also treat it as

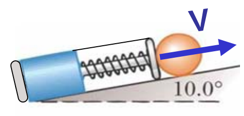
$$\frac{1}{2} k x^2 = \mu mg x + \frac{1}{2} m v^2$$

$$\frac{1}{2} \times 500 \times 0.05^2 = 0.35 \times 2 \times 9.8 + \frac{1}{2} \times 2 \times v^2$$

$v =$ 0.53 m/s

Example. The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm. The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

Answer:



Initial energy stored in the spring

$= \frac{1}{2} k x^2 = \frac{1}{2} \times 1.20\ \text{N}/(10^{-2}\text{m}) \times (5\ \text{cm} \times 10^{-2})^2 = 0.15\ \text{J}$

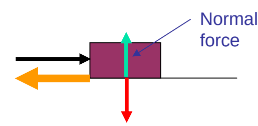
This 0.15 J is used to (i) move the ball up the slope of 10° for a vertical distance of 5 cm x $\sin(10°)$ = 0.87 cm, and

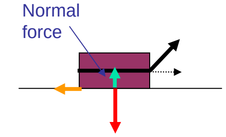
(ii) provide the ball with a muzzle speed of v. So we have

$0.1\ \text{kg} \times 9.8\ \text{m/s}^2 \times 0.0087\ \text{m} + \frac{1}{2} \times 0.1\ \text{kg} \times v^2 = 0.15\ \text{J}$

v = 1.68 m/s

Example. You need to move a heavy crate by sliding it across a flat floor with a coefficient of sliding friction of 0.2. You can either push the crate horizontally or pull the crate using an attached rope. When you pull on the rope, it makes 30° angle with the floor. Which way should you choose to move the crate so that you will do the least amount of work? How can you answer this question without knowing the weight of the crate or the displacement of the crate?

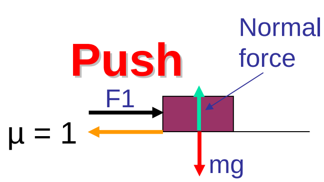




Answer:

When pushing the crate with a force parallel to the ground, the force of friction acting to impede its motion is proportional to the normal force acting on the crate—in this situation, the normal force is equal to the crate's weight. When pulling the crate with a rope angled above the horizontal, the normal force on the crate is less than its weight—the force of friction is therefore reduced. To keep the crate moving across the floor, the applied force in the parallel direction must be greater than or equal to the force of friction—pulling on the rope therefore requires a smaller parallel applied force. The work done in moving an object is equal to the product of the displacement through which it has been moved and the force component parallel to the direction of motion. The applied force component parallel to the ground is smaller when pulling the crate with the rope - thus, the work done to move the crate with the rope must be lesser, regardless of the weight of the crate or the displacement.

Push



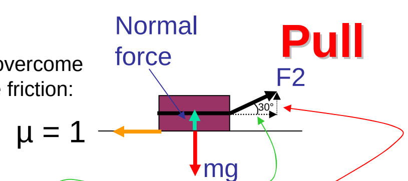
To overcome the friction:
F1 = μmg

Work done = F1 x d

What if you pull at 0 °, i.e., in parallel with the surface?

Pull

To overcome the friction:



$F2 \cos 30° = \mu\,(mg - F2 \sin 30°)$

$F2 \cos 30° + \mu F2 \sin 30° = \mu\, mg$ (μ = 1)

$F2\,(0.866 + 0.5) = \mu\, mg$

$F2 = 0.732\, \mu mg = 0.732\, F1$ (μ = 1)

Work done = (F2 cos 30°) x d

= 0.732 F1 x 0.866 x d

= 0.634 F1 x d

The mechanical advantage is

$$\frac{F1}{F2} = \frac{F1}{0.732F1} = 1.37$$

Conservation of Energy

- ***Energy is conserved***
  - This means that energy cannot be created or destroyed
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer

Power

- The time rate of energy transfer is called ***power***
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

when the method of energy transfer is work

Instantaneous Power

- The ***instantaneous power*** is the limiting value of the average power as Δt approaches zero

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

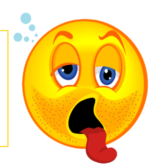
- This can also be written as

$$P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$$

Power = force x velocity

Example. **Mike applied 10 N of force over 3 m in 10 seconds. Joe applied the same force over the same distance in 1 minute. Who did more work?**

1. Mike 2. Joe
3. Both did the same work



Answer:

Both exerted the same force over the same displacement. Therefore, both did the *same amount of work*. Time does not matter for determining the work done.

Example. **Mike performed 5 J of work in 10 secs. Joe did 3 J of work in 5 secs. Who produced the greater power?**

1) **Mike produced more power**
2) **Joe produced more power**
3) **both produced the same amount of power**

Answer:

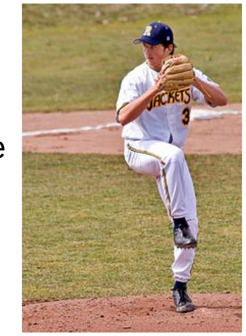
Because power = work / time, we see that Mike produced 0.5 W and Joe produced 0.6 W of power. Thus, even though Mike did more work, he required twice the time to do the work, and therefore his power output was lower.

Example. **Engine #1 produces twice the power of engine #2. Can we conclude that engine #1 does twice as much work as engine #2?**

Answer:

No!! We cannot conclude anything about how much work each engine does. Given the power output, the work will depend upon how much time is used. There are three possibilities: less, equal, more.

Example. A typical 10-year-old pitcher can throw a baseball at 70 km/h, but only a few professional athletes have the extraordinary strength needed to throw a baseball at twice that speed. Why is it so much harder to throw the baseball only twice as fast?



Answer:

Kinetic Energy = $\frac{1}{2}mv^2$.

So the baseball requires quadrupling ($\frac{1}{2}m(2v)^2$) the energy transferred to it by the professional pitcher.

The time require to deliver this energy is in one-half the time because the velocity of the ball is doubled. So the professional pitcher needs to produce **eight times as much power** as that from the 10-year-old kid ($P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$). ($dW$: x 4; dt: x 0.5)

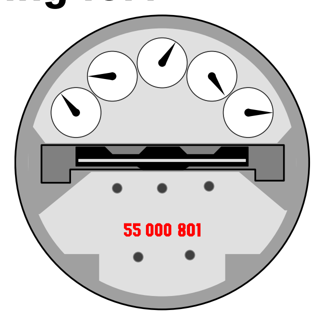
Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second
    = ((kg x m/s^2) x m) / s = 1 kg · m^2 / s^3
- A unit of power in the US Customary system is horsepower
  - 1 hp = 746 W
- Units of power can also be used to express units of work or energy
  - **1 kWh** = (1000 W)(3600 s) = 3.6 x10^6 J

Not 1 kilo-watt per hour

Energy transferred in 1 hr at a constant rate

Example. **When you pay the electric company by the kilowatt-hour, what are you actually paying for?**



Answer:

1) current
2) Voltage
3) energy
4) power
5) none of the above

Example. An energy-efficient light bulb, taking in 28.0 W of power, can produce the same level of brightness as a conventional bulb operating at power 100 W. The lifetime of the energy efficient bulb is 10 000 h and its purchase price is $17.0, whereas the conventional bulb has lifetime 750 h and costs $0.420 per bulb. Determine the total savings obtained by using one energy-efficient bulb over its lifetime, as opposed to using conventional bulbs over the same time period. Assume an energy cost of $0.080 per kilowatt-hour.

Answer:

$energy = power \times time$

For the 28.0 W bulb: (Energy efficient bulb)

$$\text{Energy used} = (28.0\ \text{W})(1.00 \times 10^4\ \text{h}) = 280\ \text{kWh}$$

$$\text{total cost} = \$17.00 + (280\ \text{kW h})(\$0.080/\text{kW h}) = \$39.40$$

For the 100 W bulb: (Conventional bulb)

$$\text{Energy used} = (100\ \text{W})(1.00 \times 10^4\ \text{h}) = 1.00 \times 10^3\ \text{kWh}$$

$$\#\ \text{bulb used} = \frac{1.00 \times 10^4\ \text{h}}{750\ \text{h/bulb}} = 13.3$$

$$\text{total cost} = 13.3(\$0.420) + (1.00 \times 10^3\ \text{kW h})(\$0.080/\text{kW h}) = \$85.60$$

$$\text{Savings with energy-efficient bulb} = \$85.60 - \$39.40 = \boxed{\$46.20}$$