PC1221 Fundamentals of Physics I

Lectures 15 and 16

Potential Energy

A/Prof Tay Seng Chuan

1

Ground Rules

- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

2

Isolated System

An <u>isolated system</u> is one for which there are no energy transfers across the boundary. The energy in such a system is conserved , i.e., at anytime the sum is a constant but its form can change in part or in whole. E.g., a block sliding across a frictionless table is moving in an isolated system. If there is friction on the table (rough surface), the block is not sliding in an isolated system any more.

3

Conservative Forces
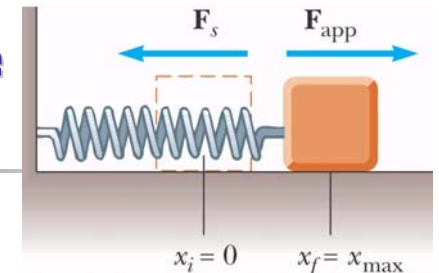
Conservative forces have these two equivalent properties:

- The work done by a conservative force on a particle moving between any two points is **independent** of the path taken by the particle.

- The work done by a conservative force on a particle moving through any closed path is zero.

The gravitational force (free fall) is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another example.

4

Potential Energy

- Potential energy is the energy associated with the configuration of a system of objects that exert forces on each other. Eg, if you stand on top of Kent Ridge Hill, your potential energy is larger than what you have now in this lecture theatre at a lower ground.
  - When conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy.
   - This is **Conservation of Mechanical Energy.**
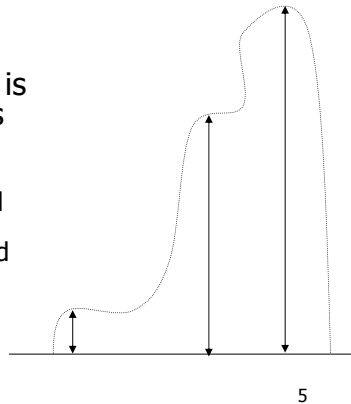
System Example

- This system consists of Earth and a book
- An applied force say from your hand does work on the system by lifting the book through $\triangle y$.
- The applied force is upward and the magnitude is mg. The work done by the applied force is mg$(y_f - y_i)$.
- At position y_f, the book has stored a higher energy. Why? The book can later fall back to y_i with a higher speed. This energy storage mechanism is called *potential energy.*
- Potential energy of any object is equal to its weight multiplied by its height from a zero potential level, i.e., ***mgh***. The object (thus the height) can be above or below the zero potential level.

Zero potential level
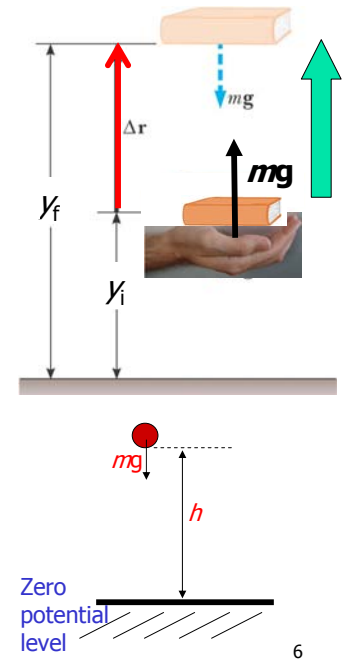
Gravitational Potential Energy

- The gravitational potential energy depends only on the vertical height of the object above Earth's surface
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero
  - The choice is arbitrary because you normally need the ***difference*** in potential energy, which is independent of the choice of reference configuration. Therefore the choice of reference frame is not important. E.g., the distance between a and b is always the same regardless of its reference with respect to X or Y.

Zero potential level
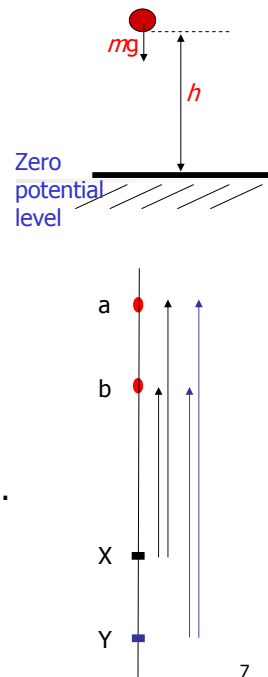
Systems with Multiple Particles

- We can extend our definition of a system to include multiple objects
- The force can be internal to the system
- The kinetic energy of the system is the algebraic sum of the kinetic energies of the individual objects

Conservation of Mechanical Energy

- The mechanical energy of a system is the algebraic sum of the kinetic energy (K) and potential energy (U_g) in the system
  - $E_{mech} = K + U_g$
- The statement of Conservation of Mechanical Energy for an isolated system (An isolated system is one for which there are no energy transfers across the boundary) is

$$K_f + U_f = K_i + U_i$$

(i.e., final sum = initial sum)

Conservation of Mechanical Energy, example (descending)

- Look at the work done by gravitational force on the book as it falls from some height to a lower height

$$W_{done\ by\ GF\ on\ book} = (-mg) \times (y_f - y_i)$$
$$= -(mg \times (y_f - y_i))$$

- However, the work done on the book is equal to the change in the kinetic energy of the book.
- So when the book is concerned,

$$\Delta K = -(mgy_f - mgy_i) = -\Delta U_g$$

Gain in kinetic energy in system   Loss in potential energy in system

- $\Delta K + \Delta U_g = 0$ (Conservation Law of Mechanical Energy in an Isolated System)
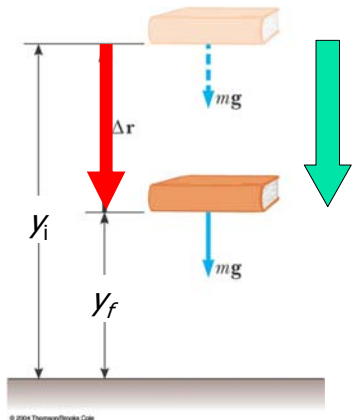
Elastic Potential Energy

- **Elastic Potential Energy** is associated with a spring
- The force the spring exerts (on a block, for example) is $F_s = -kx$
- The work done by an external applied force on a spring-block system is
  - $W = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$ (x is measured from equilibrium position.)
  - The work is equal to the difference between the initial and final values of an expression related to the configuration of the system
- This expression is the elastic potential energy: $U_s = \frac{1}{2} kx^2$, where x is the distance from the natural position.
- The elastic potential energy can be regarded as the energy stored in the deformed spring.
- The stored potential energy can be converted into kinetic energy.
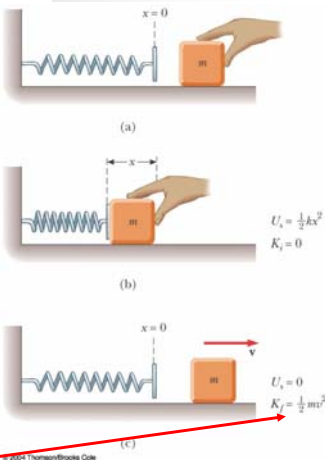
Elastic Potential Energy
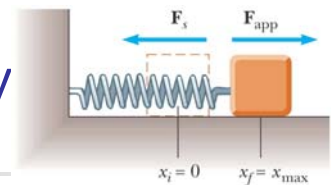
- The elastic potential energy (U) stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$)
  - The energy is stored in the spring only when the spring is stretched or compressed
- The elastic potential energy is a maximum when the spring has reached its maximum extension or compression
- The elastic potential energy ($U_s = \frac{1}{2} kx^2$) is always positive because x^2 will always be positive

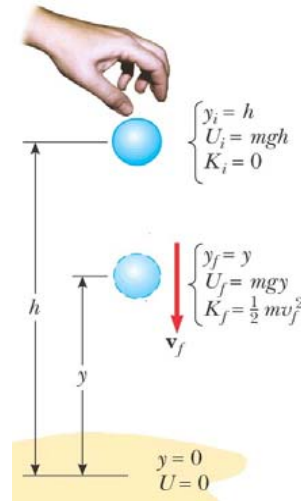# Conservation of Energy, Example 1 (Drop a Ball)

- Initial conditions:
  - $E_i = K_i + U_i = mgh$
  - The ball is dropped from rest, so $K_i = 0$
- The configuration for zero potential energy is the ground, i.e., when the ball hits the ground its potential energy will become 0
- Conservation rules applied at some point y above the ground gives
  - $\frac{1}{2} mv_f^2 + mgy = mgh$

$$\begin{cases} y_i = h \\ U_i = mgh \\ K_i = 0 \end{cases}$$

$$\begin{cases} y_f = y \\ U_f = mgy \\ K_f = \frac{1}{2} mv_f^2 \end{cases}$$

v_f

$y = 0$
$U = 0$

Example. Andrey Silnov of Russia won the men's high jump at the Beijing Olympics 2008. After his initial horizontal run, he jumped at an angle of 100° from the ground with a velocity was 6.87 m/s. How high had his center of mass move up vertically as he made the jump?
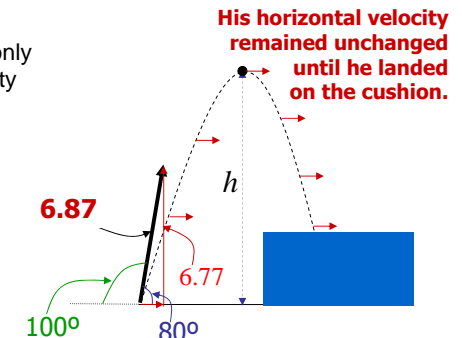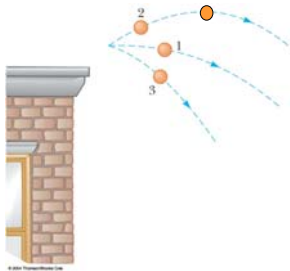
Answer:

From leaving ground to the highest point, only the kinetic energy due to the vertical velocity (6.87 x sin 80° = 6.77) was converted to potential energy, i.e.,

$$mgh = \frac{1}{2} \times m \times 6.77^2$$

$$h = \frac{6.77^2}{2 \times 9.80} = 2.34m$$

His horizontal velocity remained unchanged until he landed on the cushion.

6.87

h

6.77

100°   80°

14

Example. Three identical balls are thrown from the top of building all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal. Neglecting air resistance, rank the **speeds** of the balls at the instant each hits the ground
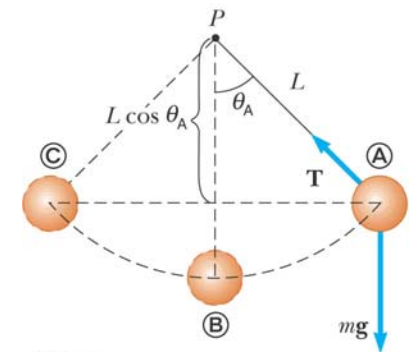
Answer:

In actual case, all the 3 speeds will be the same when the balls hit the ground. This is because the initial positions of the balls are the same (so the potential energies are the same), and the initial speeds are the same (so the kinetic energies are the same). When the balls hit the ground, the total mechanical energy (potential + kinetic) for each ball is converted to kinetic energy.

15

Conservation of Energy, Example 2 (Pendulum)

- As the pendulum swings, there is a continuous change between potential and kinetic energies
- At A, the energy is potential
- At B, all of the potential energy at A is transformed into kinetic energy
  - Let zero potential energy be at B
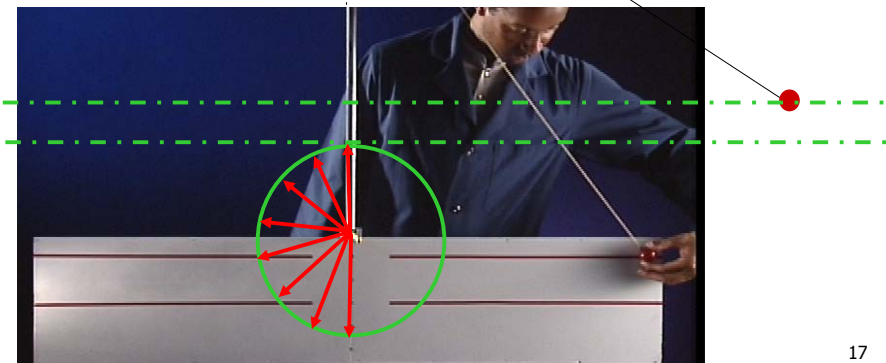- At C, the kinetic energy has been transformed back into potential energy

P

$L \cos \theta_A$ θ_A L

C   T   A

B mg

16

Top-left (slide 17)

What if we want the ball to swing at least half a circle about the pole? How?

l: the length of the string

r: radius of the circle centered at the pole

Answer:

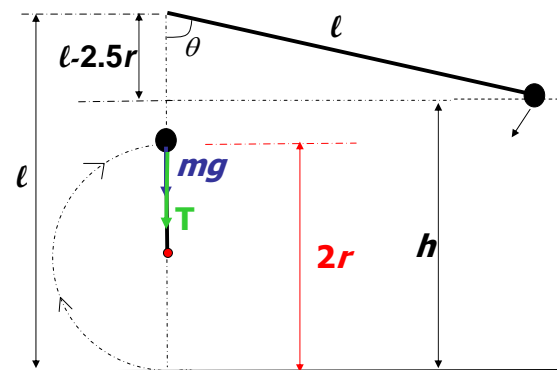$$\cos\theta = \frac{l-2.5r}{l}$$

θ

Top-right (slide 18)

Just clear the half circle

$$mg + T = \frac{mv^2}{r}$$

The energy needed to fulfill the journey

$$mgh = \frac{mv^2}{2} + mg(2r)$$

$$h = \frac{v^2}{2g} + 2r$$

$$= \frac{gr}{2g} + 2r$$

$$= \frac{5r}{2} = 2.5r$$

ℓ-2.5r

θ

ℓ

ℓ

mg

T

2r

h

$$\cos\theta = \frac{l-2.5r}{l}$$

Bottom-left

Example. Jane, whose mass is 50.0 kg, needs to swing across a river (having width D) filled with man-eating crocodiles to save Tarzan from danger. She must swing into a wind exerting constant horizontal force **F**, on a vine having length L and initially making an angle θ with the vertical. Taking $D = 50.0$ m, $F = 110$ N, $L = 40.0$ m, and $\theta = 50.0°$.
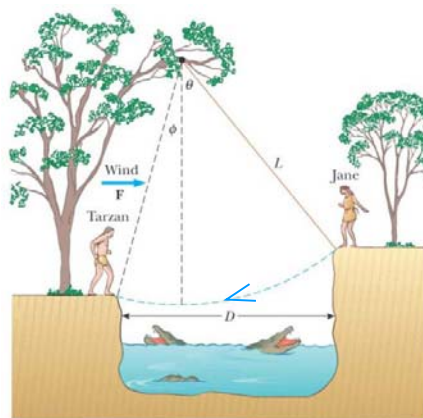
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(a) With what minimum speed must Jane begin her swing in order to just make it to the other side?

(b) Once the rescue is complete, Tarzan and Jane must swing back across the river. With what minimum speed must they begin their swing? Assume that Tarzan has a mass of 80.0 kg.

Bottom-right

(a) With what minimum speed must Jane begin her swing in order to just make it to the other side? (reverse swing ball)

0 potential level

110 N

Wind **F**

Tarzan

40.0 m L

Jane

θ

ϕ

50 m

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Answer:

$$D = L\sin\theta + L\sin\phi$$
$$50.0\text{ m} = 40.0\text{ m}\,(\sin 50° + \sin\phi)$$
$$\Rightarrow \phi = 28.9°$$
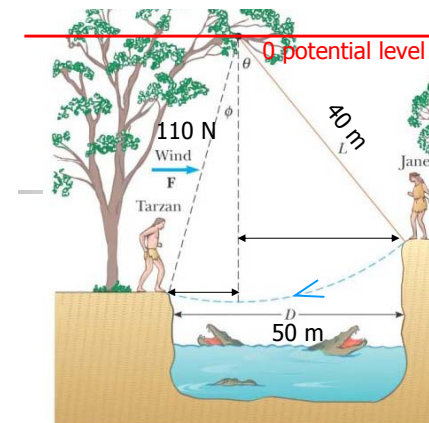
initial KE + initial PE – workdone in overcoming the wind force

= final PE at destination

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) - FD = mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg } v_i^2 + 50\text{ kg}\left(9.8\text{ m}/s^2\right)(-40\text{ m }\cos 50°) - 110\text{ N }(50\text{ m}) = 50\text{ kg}\left(9.8\text{ m}/s^2\right)(-40\text{ m }\cos 28.9°)$$

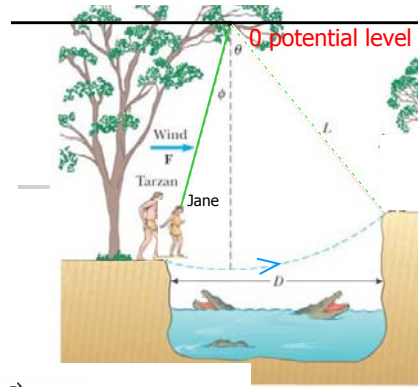$$\frac{1}{2}50\text{ kg } v_i^2 - 1.26\times 10^4\text{ J} - 5.5\times 10^3\text{ J} = -1.72\times 10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m}/s}$$

(b) Once the rescue is complete, Tarzan and Jane must swing back across the river. With what minimum speed must they begin their swing? Assume that Tarzan has a mass of 80.0 kg.

Answer:

initial KE + initial PE + workdone by the wind force

= final PE at destination

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD = mg(-L\cos\theta)$$

$$\frac{1}{2}130 \text{ kg } v_i^2 + 130 \text{ kg}\left(9.8 \text{ m}/s^2\right)(-40 \text{ m } \cos 28.9°) + 110 \text{ N }(50 \text{ m})$$

$$= 130 \text{ kg}\left(9.8 \text{ m}/s^2\right)(-40 \text{ m } \cos 50°)$$

$$\frac{1}{2}130 \text{ kg } v_i^2 - 4.46\times 10^4 \text{ J} + 5\,500 \text{ J} = -3.28\times 10^4 \text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340 \text{ J})}{130 \text{ kg}}} = \boxed{9.87 \text{ m}/s}$$

21

Conservation of Energy, Example 3 (Spring Gun)

The launching mechanism of a toy gun consists of a spring of unknown constant k. When the spring is compressed 0.12 m, the gun, when fired vertically, is able to launch a 35 g projectile to a maximum height of 20 m above the position of the projectile before firing. Neglecting all resistive forces, determine the spring constant k.
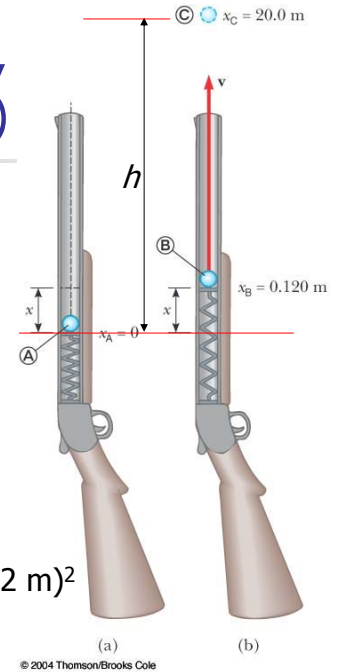
Answer: — elastic potential energy

$\frac{1}{2}\,kx^2 = mgh$ ← gravitational potential energy

$k = 2mgh / x^2$

$= 2\,(0.035 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m}) / (0.12 \text{ m})^2$

$= 953$ N/m

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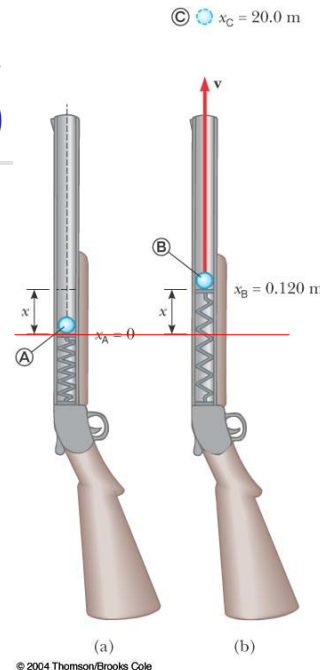
Conservation of Energy, Example 3 (Spring Gun)

Find the speed of the projectile as it moves through the equilibrium position of the spring (at X_B = 0.12 m).

Answer:

$\frac{1}{2}\,kx^2 = \frac{1}{2}\,mv_B^2$

$\frac{1}{2}$ x (953 N/m) (0.12 m)^2

$\quad = \frac{1}{2}$ x (0.035 kg)v_B^2 +

$\quad\quad$ (0.035 kg)(9.8 m/s^2)(0.12 m)

v_B = 19.7 m/s

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Nonconservative Forces

- A nonconservative force does not satisfy the conditions of conservative forces
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system

Recall that conservative forces have these two equivalent properties:

- *The work done by a conservative force on a particle moving between any two points is **independent** of the path taken by the particle.*
- *The work done by a conservative force on a particle moving through any closed path is zero.*

24

Mechanical Energy and Nonconservative Forces

- In general, if friction is acting in a system:
  - $\Delta E_{mech} = \Delta K + \Delta U = -f_k d$
  - Difference in signs is due to the loss and gain in the energy
  - ΔU is the change in all forms of potential energy
  - If friction is zero, this equation becomes the same as Conservation of Mechanical Energy, $\Delta E_{mech} = 0$, i.e., no change in mechanical energy.

Nonconservative Forces, cont

- The work done against friction is greater along the brown path than along the blue path

- Because the work done depends on the path, friction is a nonconservative force

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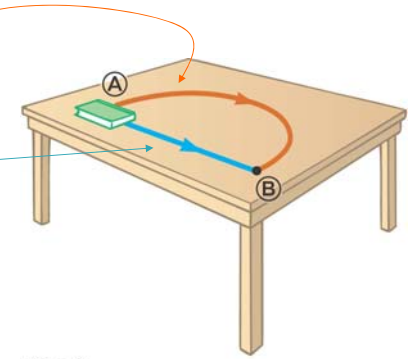
Example. You see a leaf falling to the ground with *constant speed*. When you first notice it, the leaf has initial total energy $PE_i + KE_i$. You watch the leaf until just before it hits the ground, at which point it has final total energy $PE_f + KE_f$.

How do these total energies compare?

1) $PE_i + KE_i > PE_f + KE_f$
2) $PE_i + KE_i = PE_f + KE_f$
3) $PE_i + KE_i < PE_f + KE_f$
4) impossible to tell from the information provided

Answer:

As the leaf falls, air resistance exerts a force on it opposite to its direction of motion. This force does negative work which prevents the leaf from accelerating. This frictional force is a nonconservative force, so the leaf loses energy as it falls, and its final total energy is less than its initial total energy.

Example. You throw a ball straight up into the air. In addition to *gravity*, the ball feels a force due to *air resistance*. Compared to the time it takes the ball to go up, the time it takes to come back down is:
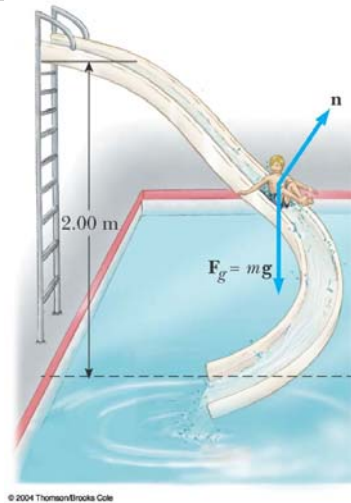
1) shorter
2) the same
3) longer
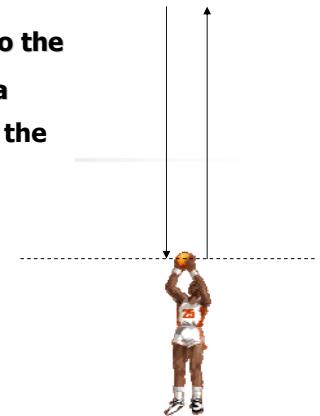
Answer:

Due to air friction, the ball is continuously losing mechanical energy. Therefore it has less KE (and consequently a lower speed) on the way down. This means it will take longer time on the way down !!
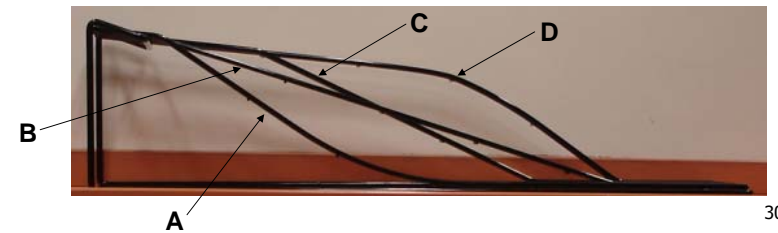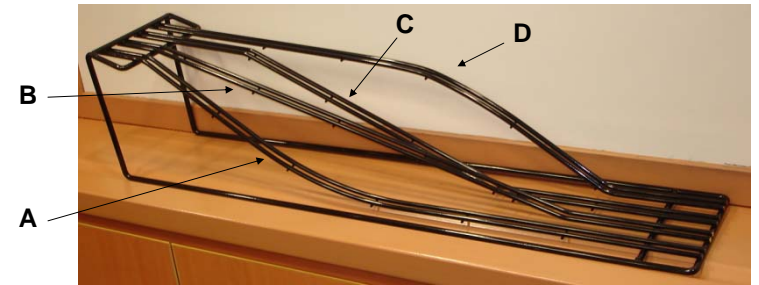
Nonconservative Forces, Example 1 (Slide)

$$\Delta E_{mech} = \Delta K + \Delta U$$
$$\Delta E_{mech} = (K_f - K_i) + (U_f - U_i)$$
$$\Delta E_{mech} = (K_f + U_f) - (K_i + U_i)$$
$$\Delta E_{mech} = \tfrac{1}{2}\, m v_f^2 - mgh$$
$$= -f_k d$$


n

2.00 m

$F_g = mg$

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We will first analyze the sequence of arrivals of these metal balls on the tracks. Which metal ball will reach the bottom first?

C   D

B

A

C   D

B

A

30

Example. Mary and John are at a water park. There are two water slides that start with the same height and end at the same height. Slide A has more gradual slope than slide B. John likes slide B better and he says he reaches at a faster speed with slide B when touching the water because he notes that he got to the bottom level in less time on slide B than on slide A as measured with his stop watch. Mary who does not carry a stop watch says she touches the water with the same speed on either slide. Who is correct and why? Both slides have negligible friction.

g sin Θ$_A$

g Θ$_A$

A

g sin Θ$_B$

g Θ$_B$

B
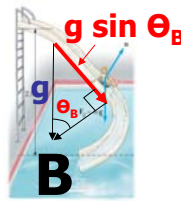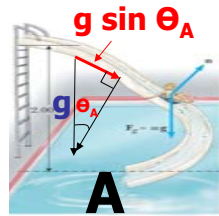
Answer:

Mary is correct. The velocity when they touch the water is the same at either slide because the vertical distance of decent is the same $mgh = \tfrac{1}{2}mv^2$.

John is partially correct. As the acceleration along the slide B **(g sin Θ$_B$)** is larger than that on slide A **(g sin Θ$_A$)**, the velocity is increased at a faster rate on slide B during the earlier duration and the fast speed continues for the rest of its journey so it takes lesser time (or is faster) to get to the bottom on slide B, but the final speeds are the same on both slides.

31

Example. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s. The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block-Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

$v_i = 8.00$ m/s

3.00 m

30°

30.0° 30°

mg
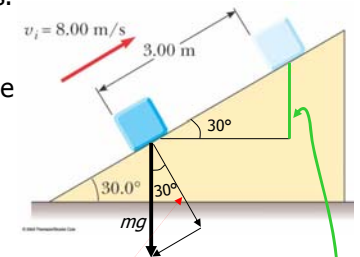
Answer:

(a) $\quad \Delta K = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right) = -\dfrac{1}{2}m v_i^2 = \boxed{-160\ \text{J}}$

(b) $\quad \Delta U = mg\left(3.00\ \text{m}\right)\sin 30.0° = \boxed{73.5\ \text{J}}$

(c) The mechanical energy converted to heat due to friction is
160 J – 73.5 J = 86.5 J. As $W = f \times d$, $f = W/d$, so

$$f = \frac{86.5\ \text{J}}{3.00\ \text{m}} = \boxed{28.8\ \text{N}}$$

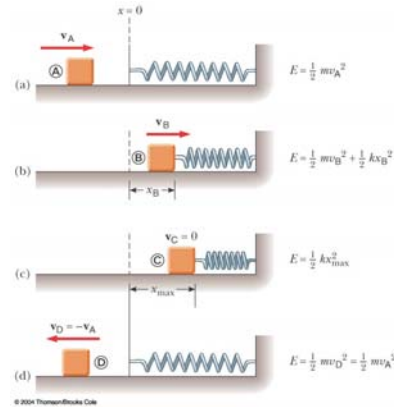(d) $\quad f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\ \text{N}$

$$\mu_k = \frac{28.8\ \text{N}}{\left(5.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\cos 30.0°} = \boxed{0.679}$$

32

## Nonconservative Forces, Example 2 (Spring-Mass)

- Without friction, the energy continues to be transformed between kinetic and elastic potential energies and the total energy remains the same
- If friction is present, the energy decreases
  - $\Delta E_{mech} = -f_k d$

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Nonconservative Forces, Example 3 (Connected Blocks)

- The system consists of the two blocks, the spring, and Earth
- Gravitational and potential energies are involved, and friction is not 0
- The kinetic energy is zero if our initial and final configurations are at rest

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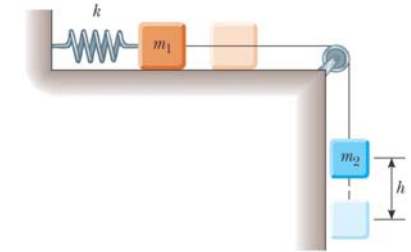
Connected Blocks, cont

- Block 2 undergoes a change in gravitational potential energy
- The spring undergoes a change in elastic potential energy
- The coefficient of *kinetic friction* can be measured
- Why is the coefficient of static friction not used?
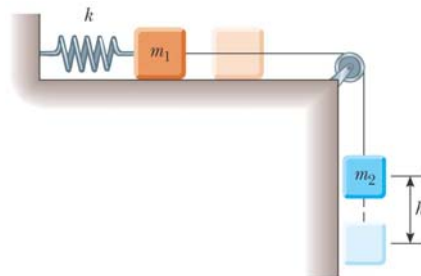
Example. A 1.00-kg object slides to the right on a surface having a coefficient of kinetic friction 0.250. The object has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has a force constant of 50.0 N/m. The object comes to rest after the spring has been compressed a distance d. The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally the object comes to rest a distance D to the left of the unstretched spring. Find (a) the distance of compression d, (b) the speed v at the unstretched position when the object is moving to the left, and (c) the distance D where the object comes to rest.
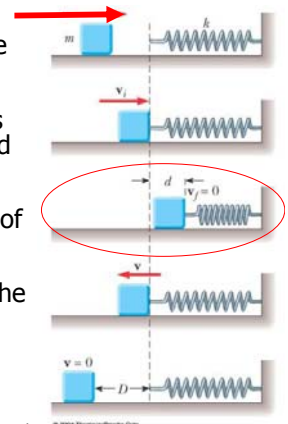
Answer:

(a) Initial amount of energy in the system is $\frac{1}{2} m v_i^2$.

This energy is given to the elastic potential energy in the spring when it is compressed and the workdone

by friction (figure 3), i.e., $\frac{1}{2} m v_i^2 = \frac{1}{2} k d^2 + \mu m g d$

$$\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m}/s^2) = \frac{1}{2}(50.0\ \text{N}/\text{m})d^2 + 0.250(1.00\ \text{kg})(9.80\ \text{m}/s^2)d$$

By the roots of quadratic equation ax² +bx + c = 0 → $d = \dfrac{[-2.45 \pm 21.25]\ \text{N}}{50.0\ \text{N}/\text{m}} = \boxed{0.378\ \text{m}}$

(b) the speed v at the unstretched position when the object is moving to the left (figure 4)
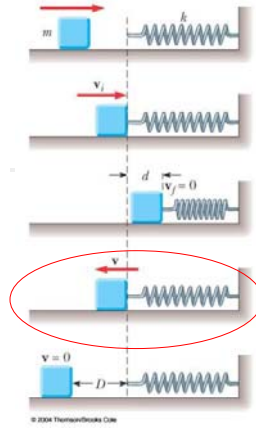
Answer:

Now the spring is at the natural position so the elastic potential energy in the spring is zero. The remaining energy left for pushing the block to the left is the initial kinetic energy less the workdone by friction where the block has travelled a distance of 2d.

$$\frac{1}{2}m v_i^2 - f(2d) = \frac{1}{2}m v^2 \qquad \times \frac{2}{m}$$

$$v = \sqrt{(3.00 \text{ m/s})^2 - \frac{2}{(1.00 \text{ kg})}(2.45 \text{ N})(2)(0.378 \text{ m})}$$

$$= \boxed{2.30 \text{ m/s}}$$

$f = \mu mg = 0.25 \times 1 \times 9.8$

$= 2.45$ N

(c) the distance D where the object comes to rest.

Answer:

As the spring is elastic, the potential energy gained and subsequently returned by the spring has a net 0 value (we assume that no heat is generated in the spring or it is negligible as the deformation is done only once).

When the (detached) object comes to rest, the initial kinetic energy is transformed to the work done by friction travelled with a distance

of (d + d + D).

$$\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2 = f(D + 2d)$$

$$D = \frac{9.00 \text{ J}}{2(0.250)(1.00 \text{ kg})(9.80 \text{ m/s}^2)} - 2(0.378 \text{ m}) = \boxed{1.08 \text{ m}}$$

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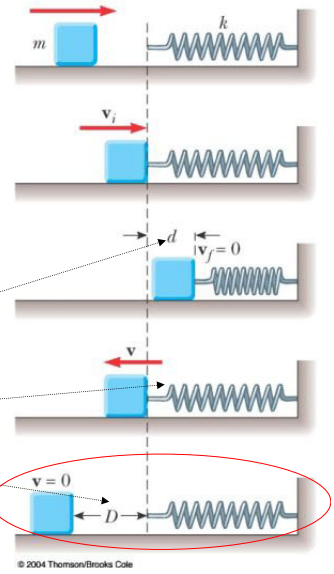
Energy Diagrams and Stable Equilibrium

- The $x = 0$ position is one of **stable equilibrium**
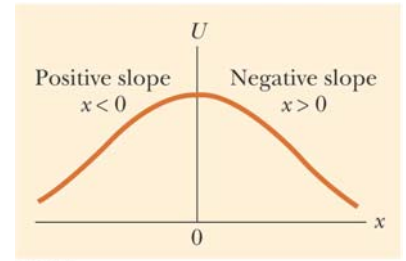- Configurations of stable equilibrium correspond to those for which potential energy $U(x)$ is a minimum

$U_s = \frac{1}{2}kx^2$
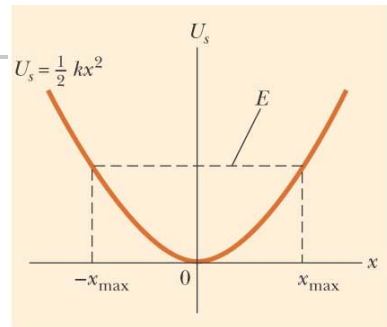
A ball in a valley is in the state of stable equilibrium.

39
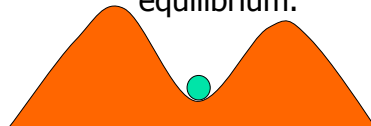
Energy Diagrams and Unstable Equilibrium

- $F_x = 0$ at $x = 0$, so the particle is in equilibrium
- For any other value of x, the particle moves away from the equilibrium position
- This is an example of **unstable equilibrium**
- Configurations of unstable equilibrium correspond to those for which $U(x)$ is a maximum

Positive slope $x < 0$ Negative slope $x > 0$

A ball on top of a hill is in the state of unstable equilibrium.

40

Neutral Equilibrium

- **_Neutral equilibrium_** occurs in a configuration when _U_ is constant over some region
- A small displacement from a position in this region will produce neither restoring nor disrupting forces

A ball on a flat ground is in the state of neutral equilibrium.
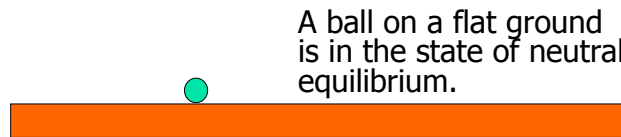
41

Example. When you push down a tumbler doll, it always move back to the up-right position. Is this tumbler doll in **neutral equilibrium**?
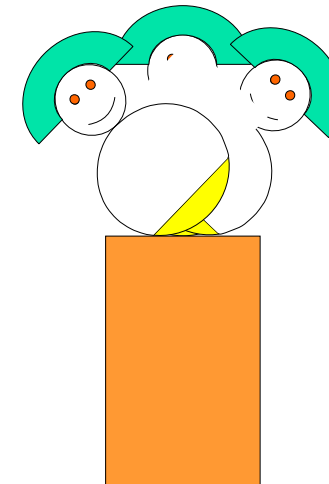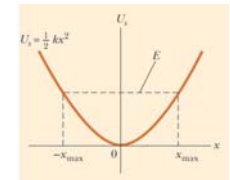
Answer:

42

Example. Similarly when you ride on this horse it always rocks. Is this horse in **neutral equilibrium**?

43

Example. When you push down a tumbler doll, it always move back to the up-right position. Is this tumbler doll in neutral equilibrium?

Answer:

No. The tumbler doll is in stable equilibrium. Its potential energy is increased whenever it is pushed down. Its weight will restore the doll back to up-right position.
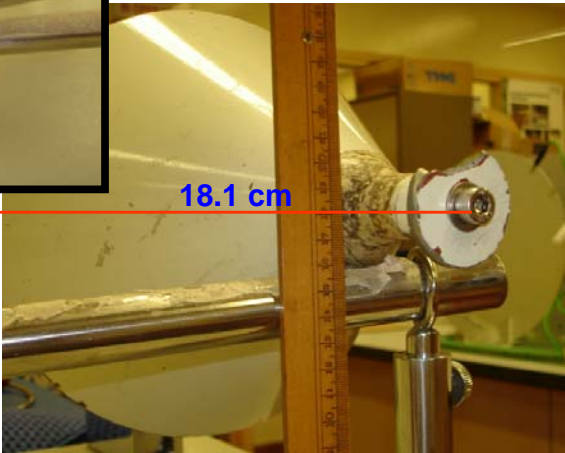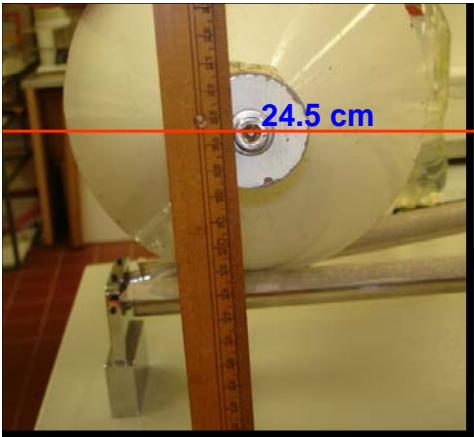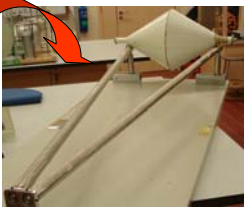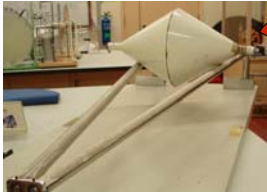

Zero potential level

44

Cone Rolling up the slope

V Track
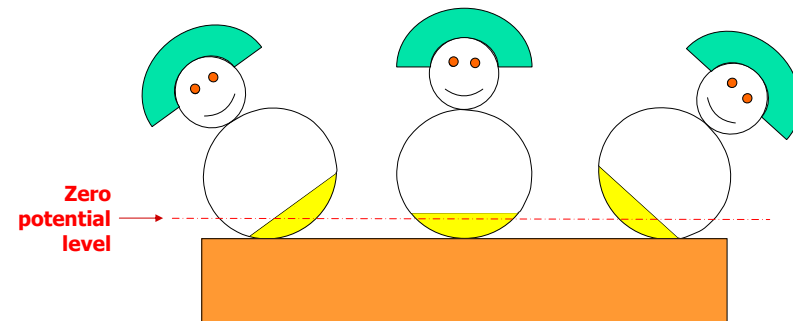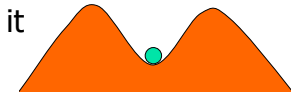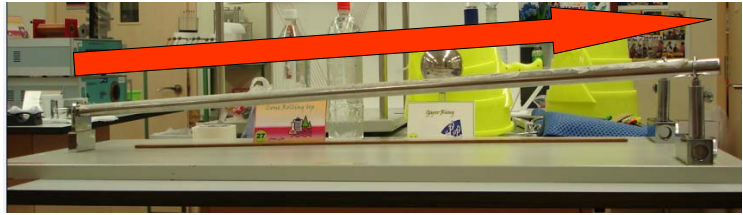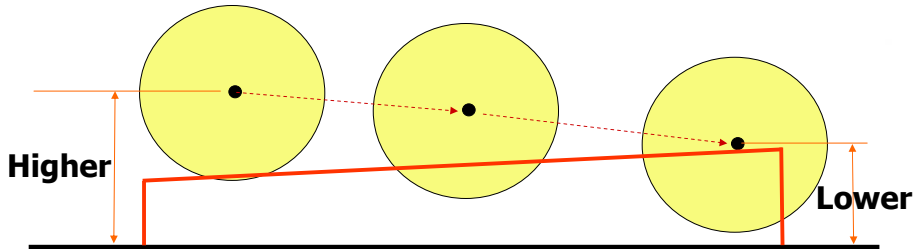

Cone Rolling up the slope

Explain the physics behind this observation.

24.5 cm

18.1 cm

Cone Rolling up the slope

Higher

Lower

Unstable equilibrium position !!

Positive slope
$x < 0$

Negative slope
$x > 0$

U

0

x

48

Panel 1 (page 49)
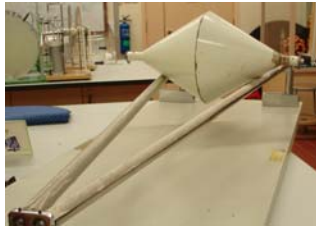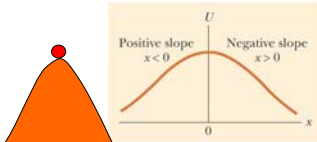
Example. A particle moves along a line where the potential energy of its system depends on its position r as graphed in Figure. In the limit as r increases without bound, $U(r)$ approaches +1 J.
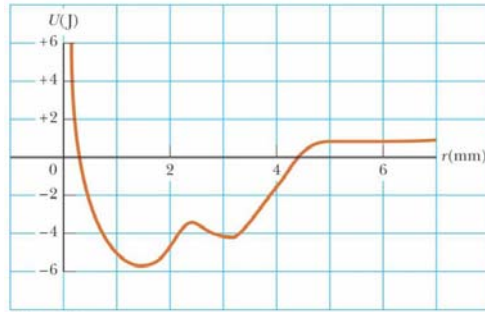

(a) Identify each equilibrium position for this particle.
Indicate whether each is a point of stable, unstable or neutral equilibrium.

Now suppose that the system (PE + KE) has energy −3 J. Determine (b) the range of positions where the particle can be found, (c) its maximum kinetic energy, (d) the location where it has maximum kinetic energy.

Panel 2 (page 50)

(a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable or neutral equilibrium.
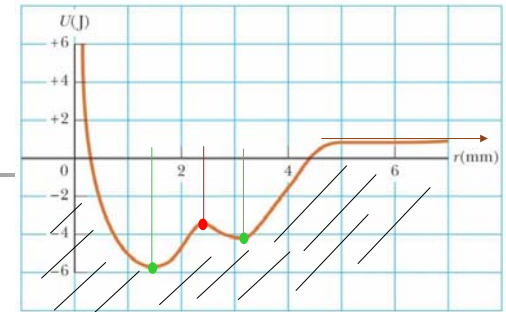
Answer:

There is an equilibrium point wherever tangent to the graph of potential energy is horizontal:

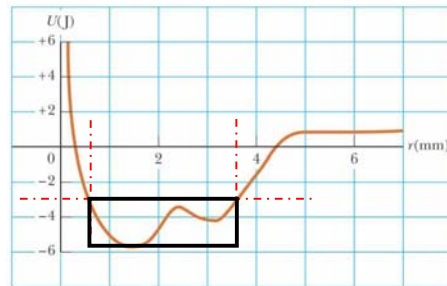At r = 1.5 mm and 3.2 mm, the equilibrium is stable.

At r = 2.3 mm, the equilibrium is unstable.

A particle moving out toward $r \rightarrow \infty$ approaches neutral equilibrium.

Panel 3 (page 51)

Now suppose that the system has energy −3 J.

Determine (b) the range of positions where the particle can be found

Answer:

If the system energy is −3 J and the system is isolated, its potential energy must be less than or equal to −3 J. Thus, the particle's position is limited to

$$0.6\,mm \le r \le 3.6\,mm$$

(c) Find its maximum kinetic energy

Answer:

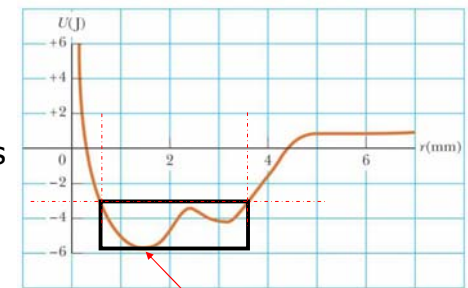$$K + U = E$$

$$K_{max} = E - U_{min}$$

$$= -3.0\,J - (-5.6\,J) = \boxed{2.6\,J}$$

Panel 4 (page 52)

Now suppose that the system has energy −3 J.

(d) Find the location where it has maximum kinetic energy

Answer:

Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\,mm}$