CZ3106 Symbolic Computing, Lab 2

Dr. Wang Jian-Sheng — For the week 27-31 Jan 03, due one week after the lab

1. Mathematica is good at calculus. Compute the following (partial) derivatives with Mathematica.

$$\frac{d}{dx}\sin x, \qquad \frac{d}{dx}x^x, \qquad \frac{\partial}{\partial y}\left(x^3 - 3xy^2\right),$$
$$\frac{d^2}{dx^2}\cos^{-1} x, \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(x+iy)^3, \quad i^2 = -1, \qquad \frac{d}{dx}f(x^2).$$

2. Differentiation can be done for any combination of algebraic expressions. But indefinite integration is much harder. Not every expression can be integrated.

(a) Try the following integration by Mathematica.

$$\int x\, dx, \qquad \int \sin x\, dx, \qquad \int (1+e^x)^{-1}\, dx.$$

(b) And some more complex (type in very very carefully):

$$\int \frac{x\left\{2\,x\,e^{3\,x^2}\left[x - \left(1 + x + 2\,x^2 + 2\,x^3\right)\,\log(1+x)\right] + \left[x^2 e^{2\,x^2} - \log^2(1+x)\right]^2\right\}}{(1+x)\,\left\{-x^2 e^{2\,x^2} + \log^2(1+x)\right\}^2}\, dx.$$

(c) Try the ratio of polynomial function. Explain the meaning of result.

$$\int \frac{6x^5 + 6x^4 - 8x^3 - 18x^2 + 8x + 8}{x^6 - 5x^4 - 8x^3 - 2x^2 + 2x + 1}\, dx.$$

(d) Try

$$\int \frac{x}{1+e^x}\, dx$$

3. Find the five (numerical) roots of the equation

$$x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1 = 0.$$

Solve the equation again with a higher precision, for example, 30 digits of accuracy.

4. Solve the set of second order equations

$$y^2 + x^2 - y - 3x = 0,$$
$$y^2 - 6xy - x^2 + 11y + 7x - 12 = 0.$$

Verify that the solutions are correct by substituting the solution back.

5. (a) Define a Mathematica function $f(x) = 1/(x^2+1)$, and make a plot of it. (b) Define the integer factorial function $n!$ in Mathematica (there are several different ways to do this). Define the greatest common divisor function $\gcd(a,b)$ of two integers; compare with build-in function `GCD[ ...]`. Generalize your gcd definition so that it works for 1, 2, 3 or *any* number of arguments.

Note: greatest common divisor integer c, of a set of integers a_1, a_2, a_3, $\ldots$, is the largest number such that c divides a_i for all i exactly. In other words, there exists integers d_i such that $a_i = cd_i$. Hint: recursive definition would be best.