CZ3106 Symbolic Computing, Lab 4

Dr. Wang Jian-Sheng   For the week 3-8 March 03, due one week after the lab

1. Develop a set of rules for Laplace transformation. The Laplace transform on a function is defined by

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt.$$

The Laplace transform has the following properties:

(a) $\int_0^\infty e^{-st}[f(t) + g(t)]\, dt = F(s) + G(s)$ (linearity).

(b) $\int_0^\infty e^{-st} c f(t)\, dt = cF(s)$ (linearity).

(c) $\int_0^\infty e^{-st} c\, dt = c/s$, where c is a constant.

(d) $\int_0^\infty e^{-st} t^n\, dt = n!/s^{n+1}$, where $n > 0$ is an integer .

(e) $\int_0^\infty e^{-st} g(t) t^n\, dt = (-1)^n d^n G(s)/ds^n$, where $n > 0$ is an integer .

(f) $\int_0^\infty e^{-st} g(t) e^{b+ct}\, dt = e^b G(s-c)$, where b and c are constants

Test your implementation by evaluating the Laplace transform of $t^2 e^{ct+b}$, where b and c are constants. Also compare your implementation with built-in function `LaplaceTransform[ ...]`.

2. A mathematical concept of the group is defined in class. Assuming a multiplication table is given, write a mathematica function to check that it is indeed a group. This includes (a) identify the identity element e, (b) for each element a, there is another element a^{-1} such that $a \cdot a^{-1} = a^{-1} \cdot a = e$, (c) and multiplication is associative, that is $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all a, b, c. Your program returns True or False for the input multiplication table. A simple exhaustive search to check the axioms should be suffice.

Test your program against the following two inputs (a) $(Z_5, *)$ defined as $(i \cdot j) = i * j \bmod 5$. (b) the table below

$\cdot$	a	b	c
a	b	c	a
b	c	b	a
c	b	a	c