CZ3106 Symbolic Computing, Tutorial 5

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1. Consider a set E generated from 1, x, e^x, and $\log x$ by addition, subtraction and multiplication. For examples, $1 + 3x$, $e^{2x} - x$, or $x \log x$ belong to E. What is the algebraic domain (ring? integral domain? or field?) of E?

2. The set G of *Gaussian integers* is the subset of the complex number C defined by

$$\mathrm{G} = \{a + b\sqrt{-1} : a, b \in \mathrm{Z}\}$$

(where we usually use the notation $\sqrt{-1} = i$, and Z is the set of all integers).

(a) Verify that G, with the standard operations of addition and multiplication of complex numbers, is an integral domain. Further, verify that G is a Euclidean domain with the valuation

$$v(a + b\sqrt{-1}) = a^2 + b^2.$$

Describe (propose) a way to compute q and r given a and b such that

$$a = bq + r, \quad v(r) < v(b), \quad a, b, q, r \in \mathrm{G}$$

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(b) What are the units in G? (Units are elements that have inverses). Give some examples of primes in G. Given some examples of associates. (If $a|b$ and $b|a$, a and b are called associates).

3.

(a) Apply Algorithm 2.1 (Euclidean algorithm) by hand to compute, in the Euclidean domain Q[x] (i.e., polynomial with rational numbers as coefficients),

$$\mathrm{GCD}(4x^4 + 13x^3 + 15x^2 + 7x + 1,\ 2x^3 + x^2 - 4x - 3).$$

(b) Repeat with extended Euclidean algorithm. Find $s(x)$ and $t(x)$ such that $\mathrm{GCD}(a(x), b(x)) = a(x)s(x) + b(x)t(x)$.

(c) Repeat the calculation in part (a) but in $\mathrm{Z}_5[x]$.