

## PC5215 Numerical Recipes with Applications

Homework problem on Monte Carlo, due Tuesday 29 Sep 2020

1. In the Buffon needle problem, prove the probability of the needle intersecting the equally spaced lines is  $P = 2L/(\pi d)$ , where  $L$  is the length of the needle, and  $d$  is strip spacing, assuming  $L < d$ .
2. A random variable  $\zeta$  with a uniform probability distribution in the interval  $[0, 1)$  is given. Work out an efficient method to generate random integers  $k \geq 0$  from  $\zeta$  with the probability distribution

$$P_k = (1-\alpha)\alpha^k, \quad k = 0, 1, 2, 3, \dots$$

where  $\alpha$  is some constant satisfying  $0 < \alpha < 1$ .

3. Suppose that  $\zeta_1$  and  $\zeta_2$  are two random variables uniformly distributed between 0 and 1, what is the probability distribution of the new variable which is a sum of the two:

$$\zeta_1 + \zeta_2 ?$$

(Hint: determine the cumulative distribution function  $F(x)$  of the new variable, then  $p(x) = dF(x)/dx$ .)

4. (a) Give the transition matrix  $W$  (an  $8 \times 8$  matrix) of a Monte Carlo simulation for a three-spin Ising system using Metropolis flip rates. A spin is chosen among the three with equal probability. The Hamiltonian (energy) of the model is

$$H(\sigma) = -J(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - h(\sigma_1 + \sigma_2 + \sigma_3), \quad \sigma_i = \pm 1,$$

where  $J$  and  $h$  are some positive constants, and assuming  $J=h$  for simplicity. (b) What is the left eigenvector of  $W$  with eigenvalue 1, i.e., find  $p$ , such that  $p = p W$ . Discuss the meaning of  $p$ .