GEK1536, Computation and Machine – ancient to modern

Midterm test, 4:50-5:50 pm, 1 Mar 2006
(Close book, 60 minutes, total 100 marks, 4 questions)
(You may use calculators)

1. [20 Marks] Answer in one or few symbols or sentences of the following:
   (a) The number four in binary representation;
   (b) Give the Hebrew symbol for one;
   (c) Give the Chinese rod numeral for seven;
   (d) Give the Egyptian hieroglyphic symbol for one hundred;
   (e) An approximate time when the sexagesimal positional system was invented;
   (f) Euclid's fifth postulate is also called ___?
   (g) The value for π accurate to the first five decimal digits after the decimal point.
   (h) The word 'algorithm' means ___ ?
   (i) The Euclidean algorithm is for computing ____?
   (j) The Roman number MCXXVI on the counting board shifted up by half a line becomes ____ ?

(a) $100_2 = 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$
(b) א
(c) _||_
(d) the rope
(e) 2000 BC to 500 BC
(f) parallel postulate
(g) $\pi = 3.14159\ldots$
(h) systematic, step by step recipes to solve a math problem
(i) greatest common divisor (GCD)
(j) ƉDCXV (55115)

2. [30 Marks] In ancient Mesopotamia, a dot positional number system was invented. The numbers from 0 to 5 or 9 are denoted by that many dots. A number is represented as … [b_5] [a_4] [b_3] [a_2] [b_1] [a_0] denoting the decimal value:

$$a_0 + b_1 10 + a_2 6\times 10 + b_3 6\times 10^2 + a_4 6^2 \times 10^2 + b_5 6^2 \times 10^3 + \cdots$$

where a runs from 0 to 9, but b goes from 0 to 5, they are represented by the corresponding number of dots. To avoid confusing, we put the dots in square brackets. Here are some of the examples:

[: .]	3
[: : : :]	8
[:] [.]	20 + 1 = 21
[: : .] [: : : : .]	59
[.] [ ] [ ]	60
[.] [: :] [: : .]	60 + 40 + 5 = 105

[.] [   ] [.] [:]   600 + 10 + 2 = 612

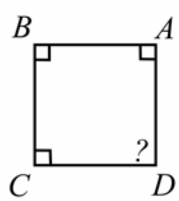
(a) Translate the following numbers into our notations:
    (i) [: .] [: :]   (ii) [:] [:] [: :]   (iii) [.] [:] [.] [:]
(b) Put the following decimal numbers into dot notations:
    (i) 24   (ii) 121   (iii) 3636

(a) (i) $3\times10 + 4 = 34$
(ii) $2\times60 + 2\times10 + 4 = 144$
(iii) $12\times60 + 12 = 732$

(b) (i) $24 = 2\times10 + 4$ *[:][::]*
(ii) $121 = 2\times60 + 1$ *[:][ ] [.]*
(iii) $3636 = 1\times3600 + 36$ *[.] [ ] [ ] [:.] [:::]*

The system is essentially the same as base-60 Babylonian.

3. [20 Marks] Consider a quadrilateral (four sided polygon) such that the angles at vertices *A*, *B*, *C* are all right angles (90 degrees).
   (a) Prove that the angle at *D* is also a right angle, assuming that the parallel postulate is valid.
   (b) If the parallel postulate is removed from Euclid's axioms, can we still prove that the angle at *D* is a right angle?



(a) *Since* $\angle B = \angle C = 90°$*, we have* $\angle B+\angle C=180°$*. By parallel postulate, AB // CD. Since AD is a line cross the parallel lines, thus* $\angle A + \angle D = 180°$*. Since* $\angle A = 90°$*, we get* $\angle D = 90°$*.*
(b) *No. We cannot decide what angle D will have if fifth postulated is removed. It is possible that* $\angle D$ *is less than* $90°$ *(in hyperbolic geometry), or greater than* $90°$ *(think of a 'square' on a sphere).*

4. [30 Marks] The Roman numerals and the counting board introduced in class only handle integers. Based on the Roman counting board, design a new counting board such that decimal fractions can also be represented.
   (a) Use your method to represent 25.75.
   (b) Use the counting board with decimal fraction to compute

25.75×1.2

Draw a sequence of key steps of your counting board configurations.

(c) Propose a method to represent your result in Roman numerals.

[Difficult to draw the graphs, so only show you in symbols and words]

(a) and (c), let us X for 0.1, C for 0.01, M for 0.001, and V for 0.5, L for 0.05, and D for 0.005, etc, using dot (.) for the decimal point, we get, for 25.75 XXV.VXXL For calculated answer 30.9 it is XXX.VXXXX

(b)The steps involve multiplying by 2, shift down to one line, and add back to the original. The answer is 30.9. The rules apply equally well for dots (pebbles) representing the integer or fractional parts.

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