NATIONAL UNIVERSITY OF SINGAPORE

PC2135 Thermodynamics and Statistical Mechanics

(Semester II: AY 2024-25)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write only your student number. Do not write your name.
2. This assessment paper contains 4 questions. It comprises 3 printed pages.
3. Students are required to answer ALL questions. The questions carry equal marks.
4. The answers are to be written in the answer book.
5. Students should write the answers for each question on a new page.
6. This is a CLOSED BOOK examination.
7. Non-programmable electronic calculators are allowed.

1. Consider a system of harmonic oscillators with energy units $\epsilon = hf = 0.025$ eV (1 eV = 1.602$\times$ 10^{-19} J).
    a. If we have $N = 3$ distinct oscillators with $q = 3$ units of energy distributed among the oscillators, how many ways, Ω, can we have? Draw figures to show the energy distributions.
    b. If we have $N = 100$ oscillators with $q = 5$ units of energy, approximately what temperature T the system is at? Answer your question in SI units of kelvin (the Boltzmann constant $k = 8.617 \times 10^{-5}$eV/K).
    c. If we put a lot of energy into the oscillators, we expect the equipartition theorem to hold. Thus, for part b with $N = 100$, for approximately what value of q equipartition holds? Find the relationship between q and the temperature T when q is sufficiently large.

2. The van der Waals equation for non-ideal gas is
$$\left[p + a\left(\frac{N}{V}\right)^2\right](V - Nb) = NkT,$$
where a and b are experimentally determined small constants. Furthermore, the internal energy of a monatomic van der Waals gas is
$$U = \frac{3}{2}NkT - a\left(\frac{N^2}{V}\right)$$
which depends on both the temperature T and volume V.
    a. Calculate the heat capacity at constant volume, C_v, as well as the heat capacity at constant pressure, C_p.
    b. Find the expression of entropy S as a function of temperature T and volume V. How does the result differ from that of the ideal gas?
    c. Give a differential equation that determines the Gibbs free energy $G = U - TS + PV$ as a function of volume V at a constant T. Solution not required.
    d. Explain what a Maxwell construction is. Draw a diagram to show such a construction and clearly explain the reason for such an operation.

Hint: the thermodynamic identity is $dU = TdS - PdV + \mu dN$.

3. Suppose we use the thermal photon gas as a working substance for a Carnot heat engine between two temperatures T_h and T_c. A photon gas has total energy from the Stefan-Boltzmann law, $U = aT^4V$, here a is some constant. The pressure is given by $PV = \frac{U}{3}$.
   a. Determine the isothermal curves (P vs V when temperature T is a constant).
   b. Determine the adiabatic curves where the entropy is a constant.
   c. Draw the Carnot cycle on the PV diagram, and label the diagram properly. Compute the efficiency $e = W/Q_h$, where W is the total work done in a cycle, and Q_h is the heat absorbed at the hot reservoir. Is photon gas more efficient or less efficient than an ideal gas as a working substance?

4. Consider a system of five particles, non-interacting, confined in one-dimensional space in a parabolic potential so that the energy levels for one particle are nondegenerate and evenly spaced at ϵ. In this problem, you will consider the allowed states for this system, depending on whether the particles are identical fermions, identical bosons, or distinguishable particles.
   a. Describe the ground state of this system for each of these three cases (that is, how the single-particle levels are occupied with the lowest possible total energy).
   b. Suppose that the system has one unit of extra energy above the ground state. Describe the allowed states of the system, for each of the three cases. How many possible system states are there in each case?
   c. Suppose that the temperature T is low compared to the energy units, ϵ/k. Estimate the heat capacity of the system for the three different cases, based on the Boltzmann factor.

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