

PC2135 Thermodynamics and Statistical Mechanics, homework 2

Due 9 Feb Thursday 11:59PM.

Problems 1.49 (15 marks, read up page 35 before tempting this problem), 2.2 (15 marks), 2.17 (15 marks), 3.5 (15 marks), 2.42 (40 marks, we'll discuss a bit in class for this problem).

Problem 1.49. Consider the combustion of one mole of H_2 with $1/2$ mole of O_2 under standard conditions, as discussed in the text. How much of the heat energy produced comes from a decrease in the internal energy of the system, and how much comes from work done by the collapsing atmosphere? (Treat the volume of the liquid water as negligible.)

Problem 2.2. Suppose you flip 20 fair coins.

- (a) How many possible outcomes (microstates) are there?
- (b) What is the probability of getting the sequence HTHHTTTHTHHHTHH-HHTHT (in exactly that order)?
- (c) What is the probability of getting 12 heads and 8 tails (in any order)?

Problem 2.17. Use the methods of this section to derive a formula, similar to equation 2.21, for the multiplicity of an Einstein solid in the “low-temperature” limit, $q \ll N$.

Problem 3.5. Starting with the result of Problem 2.17, find a formula for the temperature of an Einstein solid in the limit $q \ll N$. Solve for the energy as a function of temperature to obtain $U = N\epsilon e^{-\epsilon/kT}$ (where ϵ is the size of an energy unit).

Problem 2.42. A **black hole** is a region of space where gravity is so strong that nothing, not even light, can escape. Throwing something into a black hole is therefore an irreversible process, at least in the everyday sense of the word. In fact, it is irreversible in the thermodynamic sense as well: Adding mass to a black hole increases the black hole's entropy. It turns out that there's no way to tell (at least from outside) what kind of matter has gone into making a black hole.[†] Therefore, the entropy of a black hole must be greater than the entropy of any conceivable type of matter that could have been used to create it. Knowing this, it's not hard to estimate the entropy of a black hole.

- (a) Use dimensional analysis to show that a black hole of mass M should have a radius of order GM/c^2 , where G is Newton's gravitational constant and c is the speed of light. Calculate the approximate radius of a one-solar-mass black hole ($M = 2 \times 10^{30}$ kg).
- (b) In the spirit of Problem 2.36, explain why the entropy of a black hole, in fundamental units, should be of the order of the maximum number of particles that could have been used to make it.
- (c) To make a black hole out of the maximum possible number of particles, you should use particles with the lowest possible energy: long-wavelength photons (or other massless particles). But the wavelength can't be any longer than the size of the black hole. By setting the total energy of the photons equal to Mc^2 , estimate the maximum number of photons that could be used to make a black hole of mass M . Aside from a factor of $8\pi^2$, your result should agree with the exact formula for the entropy of a black hole, obtained* through a much more difficult calculation:

$$S_{\text{b.h.}} = \frac{8\pi^2 GM^2}{hc} k.$$

- (d) Calculate the entropy of a one-solar-mass black hole, and comment on the result.