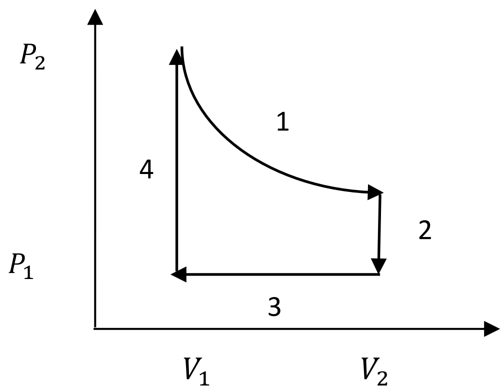
PC2135, Thermodynamics and Statistical Mechanics

Midterm test, closed book 1:45 min, 7 March 2025

Each question carries 25 marks.

1. Consider the Einstein model for vibration in solids.
    a. Let the solid A have N_A oscillators with energy $U_A = \epsilon q_A$, give the formula for the multiplicity Ω_A.
    b. Assuming the solid B is similar but with N_B oscillators and the energy units are the same as that of the solid A, so that the energy of solid B is $U_B = \epsilon q_B$. Compute the entropy S_B of the solid B using Sterling's approximation, $\ln N! \approx N \ln N - N$.
    c. If the solids A and B in parts a and b are put into thermal equilibrium such that the temperatures of the two systems are the same, $T_A = T_B$, what will be the energies of each of the systems? Here, we assume the total energy remains the same before and after equilibration. A derivation is required.

2. Consider a monoatomic ideal gas with the following loop in the P-V diagram consisting of one isothermal expansion from V_1 to V_2 starting from pressure P_2, then go vertically with an isochoric process down to pressure P_1, then go horizontally back to volume $V_1 < V_2$. Finally, close the loop by another isochoric process back to the starting point. Express your results in terms of P_1, P_2, V_1, V_2.
    a. Compute the total work W done by the gas.
    b. Compute the heat absorbed by the gas (negative if it releases heat) for each of the four processes, Q_i, $i = 1,2,3,4$.
    c. Compute the efficiency $e = W/Q_{\text{absorp}}$, here, the denominator is all the heat absorbed during the loop.



3. For a monoatomic ideal gas, we can count the multiplicity by considering the possible positions of an atom proportional to the volume V, as well as the hyper-volume in momentum space of N atoms, proportionally to $U^{3N/2}$.
   a. Write down the multiplicity for the ideal gas and give the volume and energy dependence of the entropy, ignoring the complicated factor involving the number of particles.
   b. Show that you can derive the same result in part a from the thermodynamic identity, $dU = TdS - PdV$, together with the equipartition theorem and the ideal gas law, $PV = NkT$.
   c. A liter of air, initially at room temperature (300K) and atmospheric pressure (1.013×10^5Pa), is heated at constant pressure until it is doubled in volume. Calculate the increase in its entropy ΔS during this process.

4. Consider N magnetic dipoles or spins in a magnetic field B at energy $U = \mu_B B(N - 2N_\uparrow)$, where $N_\uparrow$ the number of up spins.
   a. Write down the multiplicity of the paramagnet in terms of the total number of spins N and the number of up spins $N_\uparrow$.
   b. Assuming both N and $N_\uparrow$ are large, derive the entropy S of the system using Sterling's approximation. Eliminate $N_\uparrow$ and simplify your result as much as you can, and express the final answer in terms of U, B, N.
   c. Give the definition of chemical potential (at fixed U and B) treating each spin as a particle. Compute the chemical potential for the spins.

Physical Constants

$$
\begin{aligned}
k &= 1.381 \times 10^{-23} \text{ J/K} \\
&= 8.617 \times 10^{-5} \text{ eV/K} \\
N_A &= 6.022 \times 10^{23} \\
R &= 8.315 \text{ J/mol·K} \\
h &= 6.626 \times 10^{-34} \text{ J·s} \\
&= 4.136 \times 10^{-15} \text{ eV·s} \\
c &= 2.998 \times 10^{8} \text{ m/s} \\
G &= 6.673 \times 10^{-11} \text{ N·m}^2\text{/kg}^2 \\
e &= 1.602 \times 10^{-19} \text{ C} \\
m_e &= 9.109 \times 10^{-31} \text{ kg} \\
m_p &= 1.673 \times 10^{-27} \text{ kg}
\end{aligned}
$$

Unit Conversions

$$
\begin{aligned}
1 \text{ atm} &= 1.013 \text{ bar} = 1.013 \times 10^5 \text{ N/m}^2 \\
&= 14.7 \text{ lb/in}^2 = 760 \text{ mm Hg} \\
(T \text{ in } ^\circ\text{C}) &= (T \text{ in K}) - 273.15 \\
(T \text{ in } ^\circ\text{F}) &= \tfrac{9}{5}(T \text{ in } ^\circ\text{C}) + 32 \\
1\ ^\circ\text{R} &= \tfrac{5}{9} \text{ K} \\
1 \text{ cal} &= 4.186 \text{ J} \\
1 \text{ Btu} &= 1054 \text{ J} \\
1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
1 \text{ u} &= 1.661 \times 10^{-27} \text{ kg}
\end{aligned}
$$