SMA5231: Computing Technology and Tools

Mathematica Lab I, 2005/2006

Q1. Mathematica can do arbitrarily precise floating-point computations. Compute 101-digit approximation of the following numbers:

$$\pi, \qquad e, \qquad e^{\pi\sqrt{163}}.$$

Explore the Count instruction to find the number of times each digit occurs in the approximation.

Q2. Consider the cubic equation

$$f(x) = x^3 + 6x^2 + 9x + 1 = 0.$$

(a) Use Mathematica to plot the function to determine roughly the values of the real roots of the equation.

(b) Solve the equation symbolically. Simplify the expression as much as possible.

Q3. Use Mathematica to:

(a) Compute the exact value of the integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} xy^2 dydx$

(b) Compute the partial derivative $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(x+iy)^3$ where $i^2 = -1$.

(c) Solve the differential equation $\displaystyle\frac{dy}{dx} - y\tan x = x$ where $y := y(x)$.

Q4. Solve the equation $x = \cos(x)$ in Mathematica using your own implementation of the bisection search method. Display the iterates clearly using a Table.

Note: Suppose you want to solve $f(x) = 0$ and know two values a and b such that $f(a) < 0$ and $f(b) > 0$. If we bisect the interval at the point $\frac{a+b}{2}$ then $f(\frac{a+b}{2})$ will be 0, positive or negative. In the first case, we are done. If the value is positive, then we bisect on the interval $[a, \frac{a+b}{2}]$, else we bisect on $[\frac{a+b}{2}, b]$.