

## Singapore-MIT Alliance, CME5233 – Particle Methods and Molecular Dynamics

Tutorial 2, Monday 2:30 – 4:00, 13 Nov 2006

1. Suppose that  $\xi_1$  and  $\xi_2$  are two random variables uniformly distributed between 0 and 1, what is the probability distribution of the following new variables:

$$\xi_1 + \xi_2, \quad \xi_1 * \xi_2?$$

(Hint: determine the cumulative distribution function  $F(x)$  of the new variables, then  $p(x) = dF(x)/dx$ .)

2. Suppose we want to generate random 3D points  $(x,y,z)$  uniformly distributed on a unit sphere, e.g.,  $x^2 + y^2 + z^2 = 1$ . Give a method (or two) to do this.
3. Design a “good” simple sampling Monte Carlo method to compute the integral

$$\int_0^{\infty} \cos(x)e^{-x} dx.$$

Write down not only the estimator for the integral but also Monte Carlo error estimator.

4. Sketch or outline with pseudo-code the simple sampling Monte Carlo integration of the following two-dimensional integral,

$$I = \int_0^1 dx \int_0^x dy \sqrt{\sin(xy)}.$$

Notice that the upper limit for  $y$  is  $x$ , not 1.